Individual Longshot Preferences

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April 2016

On attitude towards risks, two stylized observations have emerged—people tend to be risk seeking when facing longshot risks, and among such people, there is a further tendency to favor bets with smaller winning probabilities for the same expected payoff. To investigate individual longshot preferences, we conduct an incentivized experiment using the fixed-odds-fixed-outcome state lotteries in China. The constructed single-prize lotteries involve winning probabilities between 10^{-1} and 10^{-5} and winning payoffs ranging from *RMB10* to *RMB10,000,000*. For lotteries with expected payoffs of *1* and *10*, we find that subjects are generally risk seeking with a substantial proportion favoring bets with intermediate winning probabilities of 10^{-1} or 10^{-3} over bets with the smallest winning probabilities of 10^{-5}. In contrast, subjects are predominantly risk averse for lotteries with expected payoffs of *100*. Moreover, a strong majority of the subjects switch from risk seeking to risk aversion as the expected payoff increases from *1* to *100* with either fixed winning probability or fixed winning payoff. Taken together, our results inform the stylized observations and shed light on models of decision making applied to longshot risks.

Keywords: longshot risk, nonexpected utility, prospect theory, rank dependent utility, betweenness, salience theory

JEL Code: C91, D8

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We gratefully acknowledge financial support from the National University of Singapore.

1. Introduction

Gambling activities in various forms from casino games, parimutuel, and sports betting to stock markets (Keynes, 1936) suggest that individuals who are ordinarily risk averse may exhibit risk seeking behavior when there is a small chance of winning a sizable prize.[1] Evidence in the racetrack betting literature suggests a tendency among bettors to overbet on longshot horses and to underbet on favorite ones, dubbed the favorite-longshot bias (Griffith, 1949). This favorite-longshot bias may also arise in state lotteries. In enhancing profitability, lottery commissions have added more numbers to the popular Lotto game resulting in much lower odds of winning but disproportionate increases in demand by the public. Hong Kong's Mark Six has progressively decreased the odds of winning over the years. In the U.S., the rules for Powerball have been gradually changed towards drastically smaller winning odds. In 2009, the odds for the jackpot were set at *1:195,249,053*. In 2012, however, the odds for the Powerball jackpot were increased slightly to *1:175,223,509*, with a decrease in the number of red balls from *39* to *35*. This development hints at a limit to the reach of the favorite-longshot bias, with lottery commissions settling for a less extreme longshot probability of winning the jackpot.

These empirical observations and a number of experimental studies suggest that decision makers are risk seeking towards small probabilities of winning sizable gains. This behavior is regarded as one of the stylized observations in risk attitude (Tversky and Kahneman, 1992). At a given expected payoff, as the winning probability gets smaller, risk attitude may diverge. On the one hand, the intuition of favorite-longshot bias suggests that the smaller the winning probability, the higher the value of the lottery. On the other hand, when the probability is sufficiently small, decision makers may find the odds of winning the lottery to be negligible.[2] The decision maker may then end up favoring a lottery with an intermediate winning probability. In this regard, Kahneman and Tversky (1979) observe that "*Because people are limited in their ability to comprehend and evaluate extreme probabilities, highly unlikely events are either ignored or overweighed…"*

This study experimentally investigates individual preferences for longshot risks when probabilities and outcomes are explicitly known. Our first question relates to the favorite-

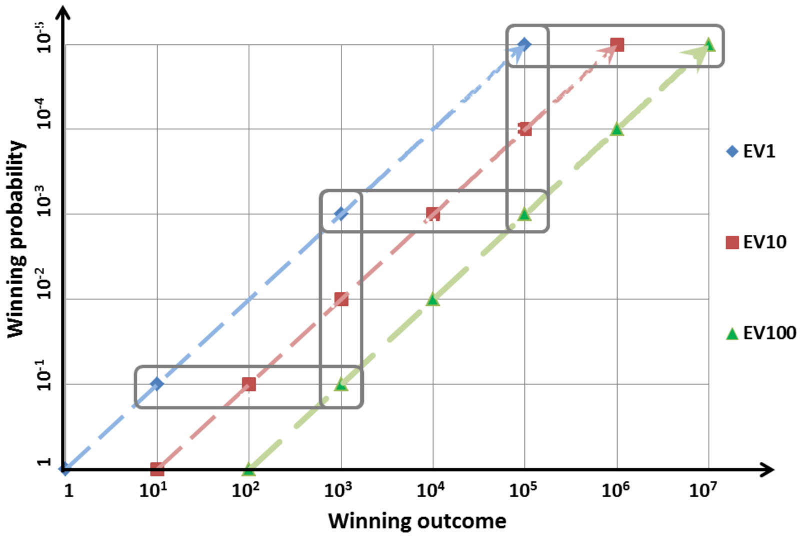
[1] In financial markets, it has been suggested that a positively skewed security can be "overpriced" and earn a negative average excess return (see, e.g., Kraus and Litzenberger, 1976), and that the preference for skewed securities might explain a number of financial phenomena such as the low long-term average return on IPO stock and the low average return on distressed stocks (Barberis and Huang, 2008).

[2] For example, decades after Bernoulli's original paper in 1728, Buffon (1777) suggests that the St. Petersburg paradox could be resolved if people ignore small probabilities. Morgenstern (1979) suggests that expected utility was not intended to model risk attitude for very small probabilities; "*the probabilities used must be within certain plausible ranges and not go to .01 or even less to .001, then be compared to other equally tiny numbers such as .02, etc.*"

longshot bias. When the expected payoff of the lotteries is fixed, do decision makers favor lotteries with progressively smaller winning probabilities or lotteries with intermediate winning probabilities? A related question is prompted by the observation that prices of lottery tickets are generally low. Are people risk seeking when the stakes are small, and are they correspondingly risk averse when the stakes are significant?

To address these questions, we investigate both theoretically and experimentally individual preferences for longshots involving small to extremely small probabilities across different levels of expected payoffs. Our experiment takes advantage of the availability of three fixed-odds-fixed-outcome state lottery products in China—1D, 3D, and 5D—paying a single prize of *RMB10* (*USD1* ≈ *RMB6.5*) with probability 10^{-1}, *RMB1,000* with probability 10^{-3}, and *RMB100,000* with probability 10^{-5}, respectively. As illustrated in Figure 1, using different combinations of the 1D, 3D, and 5D lotteries, we construct a set of single-prize lotteries with explicit winning odds ranging from 10^{-1} to 10^{-5} and explicit winning payoffs ranging from *RMB10* to *RMB10,000,000* grouped by three levels of expected payoffs—*1*, *10*, and *100*. To elicit risk attitude for equal-mean lotteries, subjects make binary choices among pairs of lotteries with the same expected payoff including receiving the expected payoff itself for sure. The experiment is conducted with 836 subjects from China.

Figure 1. Structure of lotteries used in our experiment



Note. Illustration of lotteries grouped under *EV (expected value) = 1*, *EV = 10*, and *EV = 100* involving the probabilities of 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} and winning outcomes (in RMB) of *10*, 10^2, 10^3, 10^4, 10^5, 10^6, and 10^7, using different combinations of 1D, 3D, and 5D tickets. Three sets of lotteries with the same winning probabilities are indicated by the horizontal rectangles. Two sets of lotteries with the same winning outcomes are indicated by the vertical rectangles.

We examine preference properties which are tied to our experimental design involving lotteries with a single winning outcome. One kind of properties are based on binary comparisons between lotteries with the same expected payoffs. They address our first research question on whether the decision maker exhibits (i) monotonic longshot preference by favoring a lottery with a progressively smaller winning probability, or (ii) single-peak longshot preference by favoring a lottery with an intermediate winning probability. We first examine binary choice behavior among three equal-mean lotteries with winning probabilities of 10^{-1}, 10^{-3}, and 10^{-5} and receiving the expected payoff with certainty. Besides being purely risk averse (i.e., receive the expected payoff with certainty is preferred to any of the other three lotteries), we find strong support for the tendencies of single-peak longshot preferences at expected payoffs of *1* and *10*, but not *100*. There is considerable heterogeneity in single-peak longshot preferences in terms of the individuals favoring different winning probabilities of 10^{-1}, 10^{-3}, and 10^{-5}. Interestingly, across the three levels of expected payoffs, the favored winning probability tends to increase as the expected payoff increases.

We also examine another kind of properties, based on lotteries with different levels of expected payoffs, to address our second research question on whether decision makers become more risk averse as the expected payoff increases. Comparing risk attitudes across expected payoffs, we observe a robust tendency to switch from being risk seeking to risk averse as the expected payoff increases from *1* to *100*, regardless of whether the winning probability or the winning outcome is fixed. This tendency remains when we maintain the same ratio of winning probabilities by comparing risk attitudes between pairs of lotteries with the same winning outcomes. This latter switch in risk attitude corresponds to a form of common-ratio Allais behavior as illustrated in Kahneman and Tversky's (1979) report of a preference for a *90* percent chance of winning *3,000* over a *45* percent chance of winning *6,000*, but the 'opposite' preference for a *0.2* percent chance of winning *3,000* over a *0.1* percent chance of winning *6,000*.

Our observed behavioral patterns shed light on several non-expected utility models, including rank-dependent utility (Quiggin, 1982), betweenness-conforming utility models such as weighted utility (Chew, 1983), disappointment averse utility (Gul, 1991), and salience theory (Bordalo, Gennaioli, and Shleifer, 2012), alongside expected utility. In particular, for expected utility, the Friedman-Savage (1948) reverse S-shape utility function, which includes the important case of a three-moment expected utility with variance aversion and skewness preference, cannot exhibit single-peak longshot preference. Moreover, such a decision maker cannot exhibit the switch from risk seeking to risk aversion as the payoff increases, since a reverse S-shape utility function will eventually be convex as payoff increases. By contrast, both rank-dependent utility and weighted utility can exhibit the full range of longshot related properties reflected in the observed choice behavior using a concave utility function displaying eventually decreasing elasticity coupled with their respective forms of overweighting of winning probabilities. However, in conjunction with a power utility function with constant elasticity, neither utility models can accommodate the switch from risk seeking to risk aversion as the winning outcome increases.

The paper proceeds as follows. Section 2 presents our experimental design and implementation. Properties involving longshot preferences are defined in Section 3 while Section 4 derives conditions under which different utility models may be able to exhibit specific preference properties. Section 5 presents the experimental results in terms of observed choice patterns among lotteries with the same expected payoffs and across different levels of expected payoffs. We discuss and conclude in Section 6.

2. Experimental Design

We develop an experimental design using three kinds of fixed-odds-fixed-payoff state lotteries in China known as 1D, 3D and 5D. A 1D ticket pays *RMB10* if the buyer chooses a one-digit number between *0* and *9* which matches a single winning number. Similarly, for each 3D ticket, each buyer chooses one three-digit number from *000* to *999* and wins *RMB1,000* if the number matches a single winning number. Likewise, a 5D ticket pays *RMB100,000* if the buyer's five-digit number matches the winning number. The digit lottery tickets cost *RMB2* each, and are on sale daily, including weekends, through authorized outlets by two state-owned companies. The China Welfare Lottery sells the 1D lottery and the China Sports Lottery sells both 3D and 5D lotteries. The winning numbers for each lottery are generated using Bingo blowers by independent government agents and are telecast live daily at 8 p.m. Buyers may pick their own numbers or have a computer generate random numbers at the sales outlet. Winners may cash winning tickets at the lottery outlets.

Figure 1 in the Introduction presents the parametric structure of the single-prize lotteries used in this paper. In addition to lotteries with winning probabilities of 10^{-1}, 10^{-3}, and 10^{-5} for expected payoffs of 10 and 100, we can generate lotteries with winning probabilities of 10^{-2} and 10^{-4} using different combinations of tickets. Each lottery used pays a gain amount x with probability p and pays 0 with probability $1 - p$. We denote such a lottery as (x, p) and the certainty case of $(z, 1)$ as $[z]$. For example, $(10^3, 10^{-2})$ can come from ten 3D tickets with different numbers, and $(10^5, 10^{-4})$ corresponds to ten 5D tickets with different numbers. We can similarly generate two *EV100* lotteries, $(10^4, 10^{-2})$ and $(10^6, 10^{-4})$, with these winning probabilities. Notice that this approach does not work for lotteries with expected payoffs of 1, and we are limited to using 10^{-1}, 10^{-3}, and 10^{-5} as the winning probabilities for lotteries with expected payoffs of *1*. Overall, we include four lotteries with expected payoffs of *1*, ten lotteries with expected payoffs of 10, and ten lotteries with expected payoffs of *100*. We summarize the details of these lottery products and how we generate the lotteries used in the experiment in Table A1 in Appendix A. Notice that the constructions of these lotteries are constrained by the state lotteries. For example, at expected payoffs of *1* and *10,* we are limited to winning probabilities which are not more than 10^{-1}. Each subject always chooses between two lotteries, the bulk of which have the same expected payoffs except for the four comparisons in which one choice stochastically dominates the other to test for subjects' "rationality" or attentiveness (see Table A2 in Appendix A for details).[3]

Implementation. The experiment was conducted in an internet-based setting. Running experiments online has become increasingly common in experimental economics research. For example, Von Gaudecker et al. (2008) compare laboratory and internet-based experiments, and show that the observed differences arise more from sample selection rather than the mode of implementation. Moreover, they find virtually no difference between the behavior of students in the lab and that of young highly educated subjects in the internet-based experiments. The internet-based experiment is convenient for collecting large samples, which could be helpful in conducting individual level analysis. In the experiment, each choice is displayed separately on each screen, as shown in Appendix D. We randomize the order of appearance of the 100 binary comparisons as well as the order of appearance within each comparison. At the end of the experiment, subjects answer questions about their demographics.

The potential subjects (*N = 1,282*) are Beijing-based university students whom we recruited for a large study. These subjects have previously received compensation from participating in our experiments in both classroom and online settings, so they are likely to trust us in delivering their experimental earnings. We sent email invitations followed by two reminder emails over a two-month period. Based on our survey data, the average monthly expenses of the students is about RMB1,200. We ended up with a sample of *836* subjects (*50.0* percent females; average age = *21.8*) with a high response rate of *65* percent. On average, subjects spent *19.3* minutes in the experiment. Each subject received RMB20 for participating in the experiment.

[3] One product, 2D, paying RMB98 rather than RMB100 at 1 percent chance, is used in constructing four binary comparisons to detect violations of stochastic dominance.

In order to incentivize their choices, ten percent of the subjects were randomly selected to be compensated by receiving his/her chosen lottery from a randomly selected choice out of *100* choices made. The lottery was randomly chosen in the following ways. We added the subject's birthday (year, month, and date—eight numbers in total) to get a one-digit number (0-9). If this number is the same as the sum of the "3D" Welfare lottery on Feb 28, 2013, the subject will get the additional payment. Hence, each subject has a ten percent chance of receiving the additional payment. The ten percent of the subjects who were randomly selected chose one number between *1* and *100*, which determines one decision out of the 100 decisions they made. If their choice on that round is a certain amount of money, they will receive that amount of money. Should that choice be a lottery, the experimenter will purchase the corresponding combination of lottery tickets from a state lottery store. The theoretical and empirical validity of this random lottery incentive has been a subject of debate (see, e.g., Starmer and Sugden, 1991; Wakker, 2007; Freeman, Halevy, and Kneeland, 2015, for related discussions). We adopt this incentive method in our current study because it offers an efficient way to elicit subjects' preferences, it is relatively simple, and it enables us to analyze choice behavior at the individual level.

3. Properties involving Longshot Preferences

In our design, subjects choose between pairs of equal-mean lotteries $(m/q, q)$ and $(m/r, r)$ with $q > r$, where (x, p) denotes a single-prize lottery paying x with probability p and paying 0 with probability $1 - p$. Receiving an amount x with certainty is denoted by $[x]$. We refer to a preference for $(m/q, q)$ over $(m/r, r)$, denoted by $(m/q, q) \succ (m/r, r)$, as being risk averse, and the opposite preference for $(m/r, r)$ over $(m/q, q)$, denoted by $(m/q, q) \prec (m/r, r)$, as being risk seeking. We refer to a preference for $[m]$ over $(m/q, q)$ as risk averse towards $(m/q, q)$, and a preference for $(m/q, q)$ over $[m]$ as risk seeking towards $(m/q, q)$.

Notwithstanding the prevalence of risk aversion in insurance and financial markets, it has been observed that people tend to exhibit risk seeking behavior when the winning probability is small. The generally low nominal price of a lottery ticket further suggests a general tendency for risk seeking behavior to be more widespread when the stakes are not significant. Our experiment enables us to test whether the decision maker is risk seeking for small winning probabilities of 10^{-1}, 10^{-3}, and 10^{-5} at the three levels of expected payoffs of *1*, *10*, and *100*. Here we are interested in exploring how risk attitudes may change as we vary the parameters of the lotteries.

First, we are interested in the way risk attitudes may vary when the winning probability p shrinks while the expected payoff is maintained at m. This idea is related to the favorite longshot bias at an individual level in which the decision maker will increasingly value $(m/p, p)$ as p decreases towards 0 (Chew and Tan, 2005). We state this property formally below.

Property M. A decision maker exhibits *monotonic longshot preference* at m over $(0, q]$ if $(m/q, q) \succ [m]$ and $(m/p, p) \succ (m/p', p')$ with $0 < p < p' < q$.

In our experimental setting, should the decision maker be risk seeking towards a lottery with 10^{-1} winning probability, the monotonic longshot preference property implies a preference for the lottery with 10^{-5} winning probability over the lottery with 10^{-3} winning probability, which is in turn preferred to the lottery with 10^{-1} winning probability, when all three lotteries have the same expected payoff. Alternatively, the decision maker may have a favored winning probability of p^* at expected payoff m in being increasingly risk seeking as the winning probability decreases from q to p^*, and then switch to being increasingly risk averse as the winning probability further decreases from p^*. We state this single-peak property below.

Property SP. A decision maker exhibits *single-peak longshot preference* at m over $(0, q]$ if $(m/q, q) \succ [m]$ and there is a *favored winning probability* p^* such that $(m/p, p) \succ (m/p', p')$ for $p^* < p < p' < q$ and $(m/p, p) \prec (m/p', p')$ for $0 < p < p' < p^*$.

In the limit, as p^* tends towards 0, single-peak longshot preference becomes *monotonic longshot preference*, which relates to favorite longshot bias at the individual level. In our experiment, single-peak longshot preference over $(0, 10^{-1}]$ is compatible with four choice patterns: (i) one with 10^{-1} as the favored winning probability: $(m/10^{-5}, 10^{-5}) \prec (m/10^{-3}, 10^{-3}) \prec (m/10^{-1}, 10^{-1})$; (ii) two with 10^{-3} as the favored winning probability: $(m/10^{-5}, 10^{-5}) \prec (m/10^{-1}, 10^{-1}) \prec (m/10^{-3}, 10^{-3})$ and $(m/10^{-1}, 10^{-1}) \prec (m/10^{-5}, 10^{-5}) \prec (m/10^{-3}, 10^{-3})$; and (iii) one with 10^{-5} as the favored winning probability: $(m/10^{-1}, 10^{-1}) \prec (m/10^{-3}, 10^{-3}) \prec (m/10^{-5}, 10^{-5})$. Notice that case (iii) is observationally indistinguishable from monotonic longshot preference over $(0, 10^{-1}]$.

We say that a decision maker exhibits longshot preference at m over $(0, q]$ if her preference is either monotonic or single-peak. We next investigate the potential tendency towards risk aversion when the stake in terms of expected payoff increases. We examine this tendency in three ways: (i) when the winning probability is fixed; (ii) when the winning outcome is fixed; and (iii) when winning outcomes of pairs of lotteries are fixed so that the ratio of winning probabilities remain the same. We state these properties formally below.

Property SA (*Scale aversion*). (i) The decision maker exhibits *outcome scale aversion* at probability q if there is m^* such that $(m/q, q) \succ [m]$ for $m < m^*$ and $(m/q, q) \prec [m]$ for $m > m^*$. (ii) The decision maker exhibits *probability scale aversion* at outcome x *if there is* m^* *such that* $(x, m/x) \succ [m]$ for $m < m^*$ and $(x, m/x) \prec [m]$ for $m > m^*$. (iii) The decision maker exhibits *common-ratio scale aversion* at outcomes $H > L$ if there is m^* such that $(H, m/H) \succ (L, m/L)$ for $m < m^*$ and $(H, m/H) \prec (L, m/L)$ *for* $m > m^*$.

Relatedly, the intuition behind scale aversion suggests that the favored winning probability itself would increase as the expected payoff increases. In our setting, a decision maker who is risk seeking towards lottery $(10^3, 10^{-3})$ may become risk averse towards lottery $(10^5, 10^{-3})$ from outcome scale aversion, or become risk averse towards lottery $(10^3, 10^{-1})$ from probability scale aversion. On the other hand, a decision maker who is risk seeking towards $(m/q, q)$ would need to remain risk seeking towards $(m'/q, q)$ for $m' > m$. For example, if the decision maker is risk seeking towards $(10^3, 10^{-1})$, the decision maker will also be risk seeking towards lottery $(10^3, 10^{-3})$ as well as lottery $(10, 10^{-1})$. Notice that pure risk aversion or pure risk seeking for our three levels of expected payoffs is observationally indistinguishable from outcome scale aversion or probability scale aversion.

From the definition of common-ratio scale aversion above, comparing risk attitude towards two pairs of equal-mean lotteries with the same ratio L/H of winning probabilities, i.e., $(L, m/L)$ and $(H, m/H)$ versus $(L, m'/L)$ and $(H, m'/H)$ with $m' > m$ yields four possible choice patterns: (i) risk seeking for both pairs; (ii) risk averse for both pairs; (iii) risk seeking for the lower expected payoff comparison and risk averse for the higher expected payoff comparison; (iv) risk averse for the lower expected payoff comparison and risk seeking for the higher expected payoff comparison. Expected utility is compatible with the first two patterns but not the last two, including the third pattern commonly known as the common-ratio Allais paradox and the fourth pattern referred as reverse Allais behavior. For example, being risk seeking between $(10^3, 10^{-3})$ and $(10^5, 10^{-5})$ coupled with being risk averse between $(10^3, 10^{-1})$ and $(10^5, 10^{-2})$ represents an instance of common-ratio Allais behaviour.

4. Implications of Utility Models

In this section, we investigate the conditions under which different utility models can exhibit the various properties of longshot preference. Under the expected utility model (EU), we discuss the implications under two kinds of non-concave utility functions in order to model the incidence of risk seeking behavior. As EU cannot exhibit Allais behavior arising from common-ratio scale aversion, we consider non-expected utility models including rank-dependent utility (RDU) and the betweenness class of weighted utility (WU) and disappointment aversion utility (DAU). To model longshot-related preference properties, each model applies the notion of overweighting of winning probability in conjunction with a concave utility function.

4.1. Expected utility

For a general lottery (x_i, p_i) paying outcome x_i with probability p_i, its EU is given by $\sum_{i=1}^{n} p_i u(x_i)$. Our setting concerns lotteries paying a single winning outcome with some winning probability and expected payoff. Here, in investigating preference among these lotteries at a given expected payoff of m, we denote the lottery as $(y, m/y)$ so that its EU is proportional to the slope $u(y)/y$ of the ray from the origin. To model the possibility of being risk seeking in some instances in our experiment, the utility function u in EU needs to be non-concave. For the case of u being a convex function, $u(y)/y$ is monotonically increasing in y so that it can exhibit monotonic longshot preference but not single-peak longshot preference. As it turns out, the implications of a convex utility overlap with those of a reverse S-shape utility function u (Friedman and Savage, 1948) which includes the important case of a three-moment EU model incorporating variance aversion and skewness preference (see, e.g., Golec and Tamarkin, 1998). As with the case of a convex u function, this specification can exhibit monotonic longshot preference since $u(y)/y$ is increasing for $y > y'$, once we have $u(y')/y' > u(m)/m$ at some y', but not single-peak longshot preference. Moreover, EU with a reverse S-shape u function cannot exhibit scale aversion in outcome or in probability since the specification is eventually risk seeking.

We next consider the case of an S-shape utility function (Markowitz, 1952). Here, utility is maximized when the slope of ray $u(y)/y$ equals the slope of the curve, $u'(y)$, i.e., where its elasticity $\varepsilon_u(y)$ given by $y^*u'(y^*)/u(y^*)$ equals unity, since $u(y)/y$ increases (decreases) when $\varepsilon_u(y)$ is less (greater) than one. It follows that this specification exhibits single-peak longshot preference with the favored winning probability p^*, given by m/y^*, but not monotonic longshot preference. Note that the optimal winning outcome y^* is determined by the shape of the u function, not on the expected payoff m, while the favored winning probability p^* is proportional to m. Observe that this specification can exhibit scale aversion in outcome and in probability since u is eventually concave.

4.2. Rank-dependent utility

For a general lottery (x_i, p_i) paying outcome x_i (arranged in a descending order) with probability p_i, the expression of its RDU is given by

$$\sum_{i=1}^{n}\left[\pi\left(\sum_{j=1}^{i} p_j\right) - \pi\left(\sum_{j=1}^{i-1} p_j\right)\right]u(x_i).$$

We list below several forms of π functions in the literature, each of which is initially concave and overweights small probabilities.[4]

$p^c/[p^c+(1-p)^c]^{1/c}$	Tversky and Kahneman (1992)
$\tau p^d/[\tau p^d+(1-p)^d]$	Goldstein and Einhorn (1987)
$exp\{-\beta[-ln\, p]^{\alpha}\}$	Prelec (1998)

For a lottery $(y, m/y)$ paying outcome y with probability m/y, its RDU is given by $\pi(m/y)u(y)$. The sign of its derivative with respect to y is given by that of the difference $\varepsilon_u(y) - \varepsilon_\pi(m/y)$, where the elasticity of the π function $\varepsilon_\pi(p)$ is given by $p\pi'(p)/\pi(p)$. Should $\varepsilon_\pi(m/y)$ be uniformly bounded from above by $\varepsilon_u(y)$, RDU is increasing in y giving rise to monotonic longshot preference. For example, the elasticity of a piecewise linear $\pi = a + bp$ which increases from 0 to $b/(a + b)$ can be uniformly less than the elasticity ρ of a power utility function x^ρ.

To derive the condition for single-peak longshot preference at m over $(0, q]$, it is convenient to use the winning probability p as the choice variable in the maximization problem:

$$\max_{p\in(0,q]} \pi(p)u(m/p),$$

with first-order condition given by $\varepsilon_u(m/p) = \varepsilon_\pi(p)$. A sufficient condition for the solution p^* to be optimal over $(0, q]$ is for $\varepsilon_\pi(p)$ to decrease in p while $\varepsilon_u(m/p)$ increases in p. In this case, we can further conclude that the favored winning probability p^* increases as the expected payoff m increases. This follows from applying Topkis' theorem after verifying that the cross partial derivative of $\log[\pi(p)u(m/p)]$, given by the derivative with respect to p of $\varepsilon_u(m/p)$, is nonnegative as long as ε_u is eventually decreasing. Note that both logarithmic utility and negative exponential utility have eventually decreasing elasticity and further that the latter case corresponds to constant absolute risk aversion for RDU. Several π functions have decreasing elasticities for p near 0, e.g., Tversky and Kahneman (1992) and Goldstein and Einhorn (1987).

[4] The two-parameter class of probability weighting function in Goldstein and Einhorn (1987) also appears in Lattimore, Baker, and Witte (1992). The special case where $d = 1$ reduces to the form of probability weighting function in Rachlin, Raineri, and Cross (1991) given by $(1 + \delta(1-p)/p)^{-1}$ with $\delta = 1/\tau$ being interpreted in terms of hyperbolic discounting of the odds $(1-p)/p$ against yourself winning. See Section 7.2 of Wakker (2010) for comprehensive reviews of different forms of probability weighting functions.

Note that the corresponding elasticity for Prelec (1998) is increasing for the usual case of α less than 1 when it overweights small probabilities and decreasing when α exceeds 1.

In relation to scale aversion in probability and in outcome, consider the difference $u(px)/u(x) - \pi(p)$. Observe that RDU can exhibit scale aversion with fixed probability as long as $u(px)/u(x)$ exceeds $\pi(p)$ for a sufficiently large x. RDU can also exhibit scale aversion with fixed outcome as long as $u(px)/u(x) > \pi(p)$ as p increases, which would be the case for the more usual reverse S-shape π function since $\pi(p)$ lies below the identity line for moderate probabilities.

The case of a power u function x^ρ with constant relative risk aversion merits greater attention. As discussed above, this specification can exhibit monotonic longshot preference if ε_π is bounded from above by its elasticity ρ. It can also exhibit single-peak longshot preference but the favored winning probability p^* determined by $\varepsilon_\pi(p^*) = \rho$ is fixed. In relation to scale aversion in outcome and in probability, the relevant comparison between $u(px)$ and $\pi(p)u(x)$ yields $(p^\rho - \pi(p))x^\rho$. It follows that RDU can exhibit probability scale aversion with a power u function. Yet, once this specification exhibits risk proneness at some probability and for some outcome, it would remain risk seeking at that probability regardless of the magnitude of the winning outcome. Thus, this specification is not compatible with outcome scale aversion.

4.3. Betweenness Utility

The betweenness axiom represents an important alternative to the independence axiom (Chew, 1983, 1989; Dekel, 1986). We consider two subclasses of betweenness utility, namely disappointment aversion utility (DAU) (Gul, 1991) and weighted utility (WU) (Chew, 1983). In our setting of single-prize lotteries, the DAU of (x, p) is given by $pu(x)/(p + \lambda(1\text{-}p))$. This coincides with RDU using the Goldstein-Einhorn (1987) probability weighting function with $\tau = 1/\lambda$ and $d = 1$ (see Abdellaoui and Bleichrodt, 2007). It follows that DAU exhibits longshot related preference properties which are similar to RDU as discussed in the preceding subsection. A recent model of nontransitive choice called salience theory (Bordalo, Gennaioli, and Shleifer, 2012) exhibits transitivity in our setting and has similar predictions as DAU. We discuss this model in more detail in Section 6.

For a general lottery (x_i, p_i) paying outcome x_i with probability p_i, the expression for WU is given by

$$\sum_{i=1}^{n} p_i s(x_i) u(x_i) / \sum_{i=1}^{n} p_i s(x_i),$$

where s is a positive valued weight function which equals 1 at 0. In our setting, the utility of a single-prize lottery (x, p) is given by $ps(x)u(x)/[ps(x) + 1 - p]$ so that the winning probability is overweighted with an increasing s function. As with RDU, we adopt a concave u function with eventually decreasing elasticity. Chew and Tan (2005) show that WU can exhibit monotonic longshot preference under constant absolute risk aversion with a negative exponential u function and an increasing exponential s function. More generally, monotonic longshot preference corresponds to the WU of $(y, m/y)$ being eventually increasing in y, i.e., its first derivative is positive if

$$u'(y)/u(y) > [1 - (y - m)s'(y)/s(y)]/[ms(y)+y - m],$$

for a sufficiently large y. Observe that $s'(y)/s(y) \geq 1/(y - m)$ is a sufficient condition for the above inequality to be satisfied once it holds at some outcome relative to the expected payoff m.

As with RDU, we use the winning probability p as the choice variable in the maximization problem to derive the condition for single-peak longshot preference at m over $(0, q]$:

$$\max_{p \in (0,q]} ps(m/p)u(m/p)/[ps(m/p) + 1 - p],$$

The first-order condition is given by:

$$p/(1 - p) = [1 - \varepsilon_s(m/p) - \varepsilon_u(m/p)]/[s(m/p)\varepsilon_u(m/p) - 1],$$

where $\varepsilon_s(y) = ys'(y)/s(y)$ is the elasticity of the s function. Since $p/(1 - p)$ is increasing in p, a sufficient condition for the solution p^* to be optimal is for the RHS to be decreasing over $(0, q]$. It follows that p^* increases in m. We can verify that this condition holds for $u = ln(1+x)$ with $\varepsilon_u(x) = x/(1+x)ln(1+x)$ and $s = 1 + bx^\gamma$ with $\varepsilon_s(x) = b\gamma x^\gamma/(1 + bx^\gamma)$. Note that the condition on RHS being decreasing for p near 0 holds if $\varepsilon_s(m/p)$ and the product $s(m/p)\varepsilon_u(m/p)$ are both increasing in p while the numerator and the denominator are both positive.

Relating to scale aversion in probability and in outcome, consider the difference between the utility ratio $u(px)/u(x)$ and the decision weight $ps(x)/[ps(x) + 1 - p]$ given by

$$u(px)/u(x) - ps(x)/[ps(x) + 1 - p].$$

It is apparent that WU can exhibit scale aversion at probability p as long as u and s are both bounded. In this case, $u(px)/u(x)$ tends to 1 while the ratio $ps(x)/[ps(x) + 1 - p]$ tends to $[1+(1 - p)/ps^*]^{-1} < 1$, where s^* is the limit of $s(x)$ as x increases. We can verify that WU can also exhibit outcome scale aversion when its u function is a negative exponential function, and its weight function s is a power function. To exhibit scale aversion with fixed outcome x, we can rewrite the expression for the difference as $(1 - p)u(px)/[pu(x) - u(px)] > s(x)$, which is satisfied as p increases towards 1 as long as the slope of RHS at $p = 1$ given by $u(x)/u'(x)$ exceeds $s(x)$. This latter inequality is easily satisfied with a bounded utility function, e.g., a negative exponential where $u(x)/u'(x) = (e^{\lambda x} - 1)/\lambda$.

As with RDU, we discuss the case of a power u function separately. Here, WU can exhibit single-peak longshot preference since the first-order condition determining the favored winning probability involves the product $\varepsilon_u(y)s(y)$ decreasing for a sufficiently large winning outcome y, despite $\varepsilon_u(y)$ being constant for a power u function. In terms of scale aversion, the relevant comparison yields $p^\rho - s(x)/[ps(x) + 1 - p]$. It follows that WU can exhibit scale aversion with fixed winning probability with a judicious choice of the s function, e.g., the form of s = $1 + bx^\gamma$ considered above. However, WU cannot exhibit outcome scale aversion given that s is an increasing function.

4.4. Summary

We summarize in Table 1 the conditions under which different utility models can exhibit the various longshot related properties. With a reverse S-shape u function, EU can exhibit monotonic longshot preference over small probabilities but exhibits neither aversion to scale in probability nor aversion to scale in outcome. With an S-shape u function, EU exhibits aversion to scale in probability and in outcome, and can exhibit single-peak longshot preference. However, the favored winning probability p^* yields the same winning outcome m/p^* regardless of the level of expected payoff m. Using a concave u function with eventually declining elasticity coupled with appropriate conditions on their respective approaches to overweighting of winning probabilities, RDU and WU can each exhibit monotonic as well as single-peak longshot preference (with favored winning probability increasing with expected payoff) along with scale aversion in outcome and in probability. For the limiting case of a power utility function with constant elasticity, RDU can exhibit single-peak longshot preference but the favored winning probability is independent of the expected payoff. It is compatible with probability scale aversion but not with outcome scale aversion. WU can exhibit the full range of longshot related preference properties except for outcome scale aversion given that s is an increasing function.

Table 1. Conditions on utility models to exhibit longshot preference related properties

Property	EU, Non-concave *u* function		RDU, Concave *u* function		WU, Concave *u* function	
	S-shape	*Reverse S-shape*	*Eventually decreasing u-elasticity*	*Constant u-elasticity*	*Eventually decreasing u-elasticity*	*Constant u-elasticity*
Monotonic longshot	N	Y	Y	N	Y	Y
Single-peak longshot	Y[a]	N	Y	Y[b]	Y	Y
Outcome scale aversion	Y	N	Y	N	Y	N[c]
Probability scale aversion	Y	N	Y	Y	Y	Y
Common-ratio scale aversion	N	N	Y	Y	Y	Y

Note. **a**. Winning outcome m/p^* is fixed. **b**. favored winning probability is fixed. **c**. Incompatible with an increasing s function.

5. Results

In this section, we report the observed choice behavior among lotteries with the same expected payoffs of *1*, *10*, and *100* as well as comparisons of risk attitudes elicited at different expected payoffs. An overall sense of observed behavior can be seen from Table 2, which presents the aggregate proportions of risk averse choice at the three levels of expected payoffs. At an expected payoff of *1*, subjects generally exhibit risk seeking, but are risk averse between 10^{-3} and 10^{-5}. In particular, the winning probability of 10^{-3} seems favored since $(10^{3}, 10^{-3})$ tends to be preferred to *[1]*, and preferred to $(10^{5}, 10^{-5})$. This suggests that subjects may favor winning odds of 10^{-3} at expected payoffs of *1*. At expected payoffs of *10*, subjects are generally risk seeking relative to receiving the expected payoff with certainty, but they are risk averse between pairs of lotteries in preferring lotteries with larger winning probabilities to those with smaller winning probabilities. At expected payoffs of *100*, subjects tend to be risk averse for all comparisons.

Table 2. Proportion of risk seeking choice at each level of expected payoff.

Winning probability		Proportion of risk seeking choice (%)		
Higher	Lower	EV = 1	EV = 10	EV = 100
1	10^{-1}	*75.8*	*60.2*	*19.6*
1	10^{-2}	-	*55.0*	*18.5*
1	10^{-3}	*80.3*	*55.3*	*16.0*
1	10^{-4}	-	*52.8*	*14.5*
1	10^{-5}	*79.8*	*51.9*	*14.5*
10^{-1}	10^{-2}	-	*33.3*	*24.7*
10^{-1}	10^{-3}	*61.5*	*35.2*	*17.6*
10^{-1}	10^{-4}	-	*31.3*	*19.3*
10^{-1}	10^{-5}	*63.3*	*32.3*	*17.8*
10^{-2}	10^{-3}	-	*29.7*	*28.6*
10^{-2}	10^{-4}	-	*27.5*	*24.5*
10^{-2}	10^{-5}	-	*28.5*	*24.5*
10^{-3}	10^{-4}	-	*37.8*	*38.0*
10^{-3}	10^{-5}	*40.3*	*39.1*	*32.8*
10^{-4}	10^{-5}	-	*43.8*	*41.1*

Note. Column 1 (2) presents the winning probability for the lottery with the higher (lower) probability. Columns 3, 4, and 5 display the proportions of risk seeking choice under the three levels of expected payoffs of EV = 1, EV = 10, and EV = 100 respectively.

At the aggregate level, we arrive at the sense that subjects are generally risk seeking for lotteries with small expected payoffs. At the same time, subjects exhibit a trend in switching from risk seeking to risk aversion as expected payoff increases. We organize the exposition of our results around two questions. First, when expected payoff is fixed, whether subjects have monotonic or single-peak longshot preference. Second, when expected payoff increases whether subjects switch from risk seeking to risk aversion in relation to outcome scale aversion, probability scale aversion, and common-ratio scale aversion.

5.1. Preference within the same expected payoff

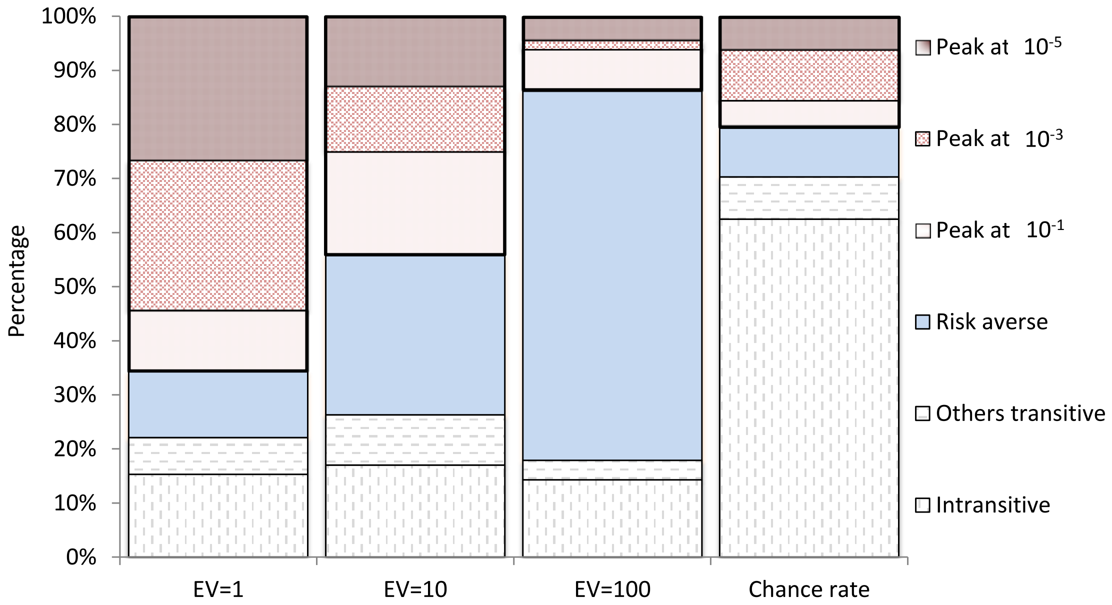
To investigate longshot preference properties, we examine subjects' choice behavior among equal-mean lotteries. There are four lotteries with winning probabilities of 1, 10^{-1}, 10^{-3}, and 10^{-5}, which are common to the three levels of expected payoffs. This yields six binary choices which in turn give rise to 64 possible choice patterns. At expected payoffs of 10 and 100, we have six lotteries with winning probabilities of 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}, giving rise to 2^{15} possible binary choice patterns which is 512 times more than is the case for lotteries with expected payoffs of 1. To compare across all three levels of expected payoffs, we focus on the 64 possible patterns from the six comparisons involving probabilities of 1, 10^{-1}, 10^{-3}, and 10^{-5}. For the 64 possible patterns, 24 patterns are consistent with transitive preferences (e.g., $10^{-3} \succ 10^{-5} \succ 1 \succ 10^{-1}$), and the rest of the 40 patterns each contains some "cycle" (e.g., $10^{-3} \succ 10^{-5} \succ 1 \succ 10^{-1} \succ 10^{-3}$).

At all three levels of expected payoffs, subjects exhibit high rates of transitive choice—*84.4* percent for expected payoff 1, *82.9* percent for expected payoff 10, and *86.7* percent for expected payoff 100—exceeding the chance rate of $24/64$ in each case ($p < 0.001$) with *61.5* percent exhibiting transitivity across all three levels of expected payoffs. This suggests that subjects' choices are mostly transitive even when the winning probabilities are extremely small at 10^{-3} and 10^{-5} regardless of the level of expected payoff. The 24 transitive patterns can be listed in an ascending order (e.g., 1053 refers to $10^{-3} \succ 10^{-5} \succ 1 \succ 10^{-1}$) (see Table A3 in Appendix A for the list). Of the 24 transitive choice patterns, thirteen are single-peak including seven for single-peak over $(0, 10^{-1}]$, four for single-peak over $(0, 10^{-3}]$, and two for single-peak over $(0, 10^{-5}]$. Six are purely risk averse—1350, 1530, 3150, 3510, 5130, and 5310. The remaining five patterns under "Others" are opposite to single-peak with 10^{-3} being worse than 10^{-1} as well as 10^{-5}.

Figure 2 displays the frequencies of single-peak longshot patterns at 10^{-5}, 10^{-3} and 10^{-1}, purely risk averse choice patterns, patterns classified under "Others" alongside those of intransitive choice (see also Table A4 in Appendix A). At expected payoffs of 1, *65.6* percent of the subjects exhibit single-peak longshot preference patterns while only *12.3* percent are purely risk averse. Among those with single-peak preferences, the observed frequency at each favored winning probability is higher than the corresponding chance rate (proportion test,

$p < 0.001$). Notice that the *23.6* percent of subjects exhibiting single-peak longshot preferences at 10^{-5} over $(0, 10^{-1}]$ is not distinguishable from the monotonic longshot preference pattern.

Figure 2. Frequencies of individual longshot preferences

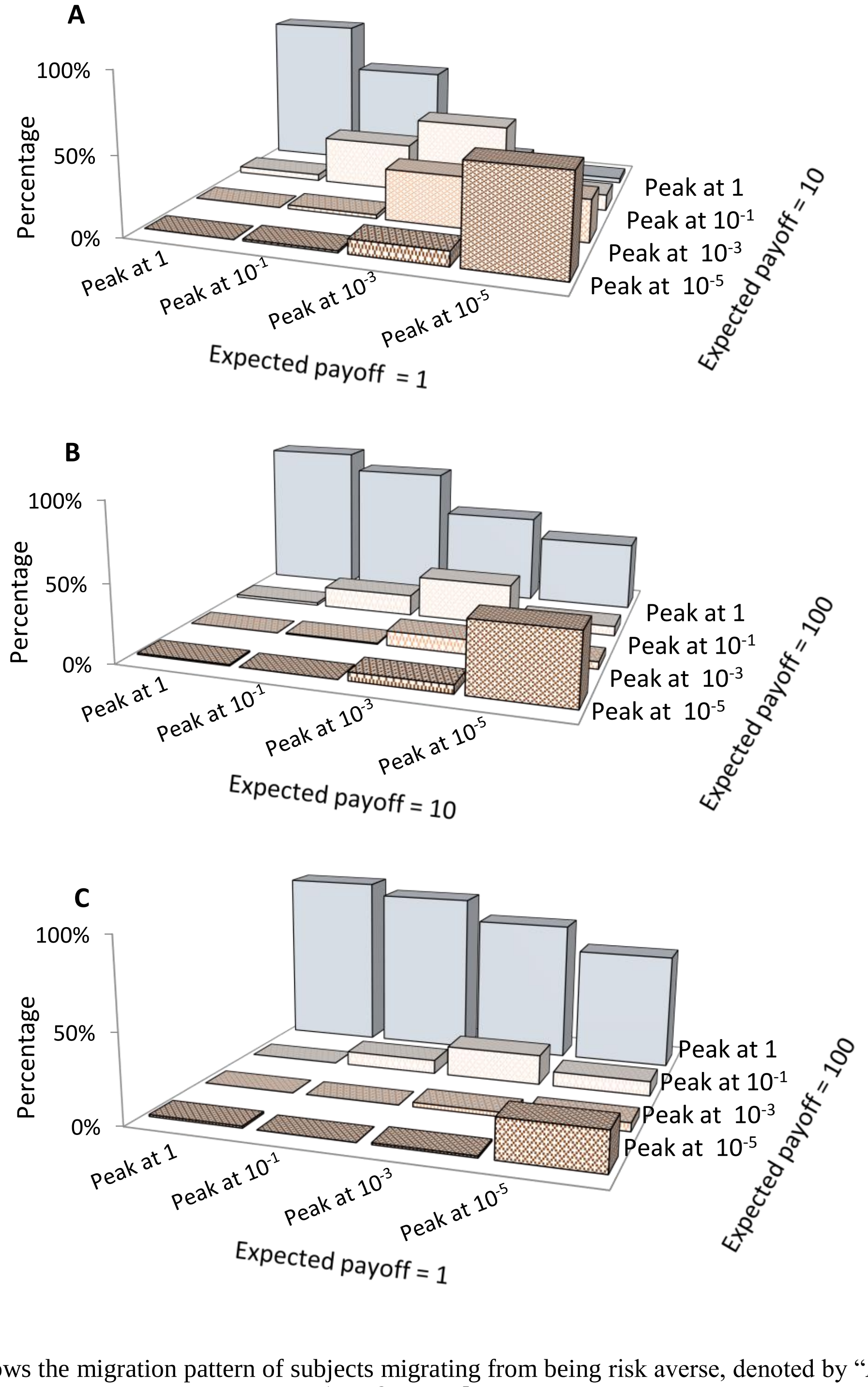


Note. This figure plots the frequencies of individual longshot preferences including single-peak at 10^{-5}, 10^{-3}, and 10^{-1} (delineated with thickened borders), purely risk averse choice, other transitive choice, and intransitive choice across expected payoffs (EV) of *1*, *10*, and *100*, compared to the chance rate.

At expected payoffs of *10*, *44.0* percent exhibit single-peak preferences while *29.7* percent are purely risk averse. The frequencies of the favored winning probabilities of 10^{-1} and 10^{-5} remain higher than the corresponding chance rate (proportion test, $p < 0.001$). At expected payoffs of *100*, a substantial majority of *68.3* percent are purely risk averse while only *13.8* percent have single-peak preferences. Here, the only significant patterns are single-peak at 10^{-1} (*7.6* percent) and the monotonic longshot preference of 0135 (*3.2* percent).

Migration pattern of favored winning probabilities as the expected payoff increases. We observe an overall migration in the favored winning probabilities at lower expected payoffs towards higher favored winning probabilities at higher expected payoffs. This observation appears to be in line with the general tendency towards risk aversion as the expected payoff increases. This tendency is apparent from Figure 3 (derived from Table A5 in Appendix A) which focuses on subjects who either are purely risk averse or have single-peak longshot preferences. Panel A shows the migration pattern across expected payoffs of *1* and *10*, Panel B shows the migration pattern across expected payoffs of *10* and *100*, and Panel C shows the migration pattern across expected payoffs of *1* and *100*.

Figure 3. Migration of favored winning probabilities across expected payoffs



Note. Panel A shows the migration pattern of subjects migrating from being risk averse, denoted by "*Peak at 1*", or having single-peak longshot preference at 10^{-1}, 10^{-3}, or 10^{-5} at expected payoffs of *1* to being risk averse, or have single-peak longshot preference at 10^{-1}, 10^{-3}, or 10^{-5} at expected payoffs of *10*. Panel B and Panel C show the corresponding migration patterns from expected payoffs of *10* to expected payoffs of *100*, and from expected payoffs of *1* to expected payoffs of *100*, respectively.

Results of Pearson's Chi-squared tests show that the occurrences of the different types across expected payoffs are not independent ($p < 0.001$). Of *165* subjects who favor 10^{-3} at expected payoffs of 1, *79* switch to 10^{-1}, *15* switch to 10^{-5}, *18* switch to being risk averse, with *53* remaining at 10^{-3}, compared with lotteries with expected payoffs of *10*. Of *86* subjects favoring 10^{-1} at expected payoffs of *1*, *57* become risk averse at expected payoffs of *10*. By contrast, for *99* subjects who are risk averse at expected payoffs of *1*, *95* remain risk averse at expected payoffs of *10*. Between expected payoffs of *10* and expected payoffs of *100*, of *287* subjects with single-peak preferences at 10^{-1}, 10^{-3}, or 10^{-5} at expected payoffs of *10*, *199* become risk averse at expected payoffs of *100*. Between expected payoffs of *1* and expected payoffs of *100*, of *439* subjects with single-peak preferences at 10^{-1}, 10^{-3}, or 10^{-5} at expected payoffs of *1*, *340* become risk averse at expected payoffs of *100*.

Summarizing, we have the following overall observation of longshot preferences involving equal-mean comparisons.

Observation 1. *Subjects exhibit a significant incidence of single-peak longshot preference at expected payoffs of 1 and 10, and are predominantly risk averse at expected payoffs of 100. For the bulk of subjects exhibiting single-peak longshot preference, the favored winning probability tends to increase with an increase in the expected payoff.*

5.2. Preferences across expected payoffs

Our design enables us to investigate small-probability risk attitudes as the expected payoff increases and examine the longshot properties of (i) outcome scale aversion and probability scale aversion; and (ii) common-ratio scale aversion.

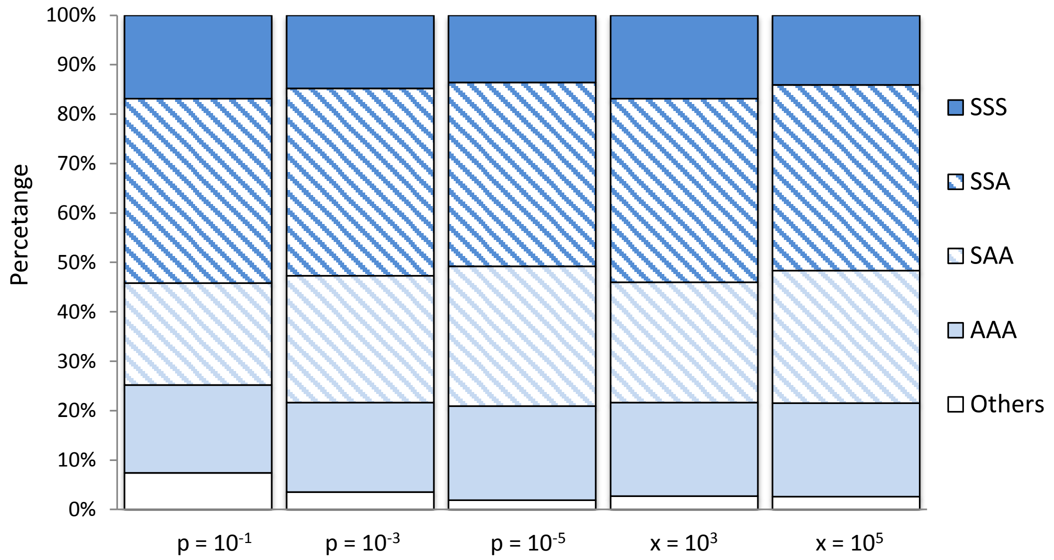
Outcome scale aversion and probability scale aversion. To examine the property of outcome scale aversion, we observe risk attitudes relative to expected payoffs across the three levels of expected payoffs while fixing the same winning probability at 10^{-1}, 10^{-3}, and 10^{-5} successively. In parallel, to examine the property of probability scale aversion, the winning outcome is fixed at *1,000* and *100,000* as the winning probability varies between 10^{-5} and 10^{-1}. In each case, there are eight possible choice patterns given that there are three comparisons between a lottery and receiving its expected payoff with certainty. Of these, four are compatible with the tendency of a switch from being risk seeking to risk averse as the expected payoff increases: (i) risk seeking across all three levels of expected payoffs; (ii) risk seeking at expected payoffs of *1* and *10*, and risk averse at expected payoff of 100; (iii) risk seeking at expected payoff of *1*, and risk averse at expected payoffs of *10* and *100*; and (iv) risk seeking across all three levels of expected payoffs. The four remaining patterns, grouped under "Others", allow for switching from risk aversion to risk seeking as the expected payoff increases. The proportion of each choice pattern is summarized in Figure 4 which is derived from Table A6 in Appendix A.

We see that the bulk of our subjects exhibit switching behavior from risk seeking to risk aversion. Specifically, the proportions of single-switch behavior—*57.9* percent (with $p = 10^{-1}$), *63.4* percent (with $p = 10^{-3}$), *65.5* percent (with $p = 10^{-5}$), *61.5* percent (with $x = 10^{3}$), and *64.4* percent (with $x = 10^5$)—are each considerably higher than their frequencies of no-switch: 32.9 percent, 32.6 percent, and 32.6 percent under outcome scale aversion and 35.8 percent and 33.0 under probability scale aversion ($p < 0.001$ in each case). Note that the proportions of "Others," given by *7.4* percent, *2.3* percent, *1.8* percent, *3.8* percent, and *2.6* percent, are each minuscule relative to the corresponding chance rate of *1/2* ($p < 0.001$ in each case).

Summarizing, we have:

Observation 2A. *Subjects exhibit outcome scale aversion and probability scale aversion in switching from risk seeking to risk averse as the expected payoff increases when* <u>*either*</u> *the winning outcome* <u>*or*</u> *the winning probability is fixed.*

Figure 4. Switching towards risk aversion as the expected payoff increases



Note. This figure plots the percentage of choice patterns across the three levels of expected payoffs including SSS (risk seeking at all three levels of expected payoffs), SSA (risk seeking at expected payoffs of *1* and *10*, and risk averse at expected payoffs of *100*), SAA (risk seeking at expected payoffs of *1*, and risk averse at expected payoffs of *10* and *100*), and AAA (risk averse at all three levels of expected payoffs), and the other four patterns, when fixing the winning probability constant as 10^{-1}, 10^{-3}, and 10^{-5}, as well as fixing the winning outcome constant as 10^3 and 10^5, respectively.

Common-ratio scale aversion. Table 3 displays 14 possible instances of common-ratio Allais behavior (relating to common-ratio scale aversion as defined in section 3) and their corresponding frequencies. The observed incidence of Allais choice pattern ranging from *13.6* percent to *27.4* percent is in line with what is reported in the literature based generally on moderate probabilities (i.e., Conlisk, 1989; Cubitt, Starmer and Sugden, 1998; List and Haigh, 2005; Huck and Muller, 2012; Nebout and Dubois, 2014). For instance, Conlisk (1989) observes that the proportion of individual-level Allais pattern is *43.6* percent for his basic treatment and *10.8* percent for his three-step treatment. List and Haigh (2005) find the proportion of such patterns to be *43* percent among student subjects and only *13* percent among professional traders. More recently, with a sample of *1424*, Huck and Muller (2012) report the proportion of Allais behavior to be *49.4* percent in a high hypothetical-payoff treatment, *19.6* percent in a low hypothetical-payoff treatment, and *25.6* percent in a low real-payoff treatment. We investigate whether the observed patterns of EU violations are systematic using Conlisk's (1989) test which takes expected utility as the null hypothesis, and compares the frequencies of Allais and reverse Allais behavior. Taking the 14 comparisons in Table 3 together, we find Allais violations to be more pronounced than reverse Allais behavior ($Z = 27.17, p < 0.001$). For these 14 comparisons, the proportion of each Allais pattern is significant at $p < 0.003$, suggesting that violations of expected utility at small probabilities are pervasive.

Table 3. Incidence of common-ratio Allais choice behavior

Item	EV = 100	EV = 10	EV = 1	Allais	Reverse Allais
1		$[10]$ vs $(10^3,10^{-2})$	$(10,10^{-1})$ vs $(10^3,10^{-3})$	*15.6%*	*9.1%*
2		$[10]$ vs $(10^5,10^{-4})$	$(10,10^{-1})$ vs $(10^5,10^{-5})$	*17.1%*	*6.6%*
3		$(10^3,10^{-2})$ vs $(10^5,10^{-4})$	$(10^3,10^{-3})$ vs $(10^5,10^{-5})$	*18.5%*	*5.7%*
4	$[10^2]$ vs $(10^3,10^{-1})$	$(10^2,10^{-1})$ vs $(10^3,10^{-2})$		*20.7%*	*7.1%*
5	$[10^2]$ vs $(10^4,10^{-2})$	$(10^2,10^{-1})$ vs $(10^4,10^{-3})$		*20.9%*	*4.3%*
6	$[10^2]$ vs $(10^5,10^{-3})$	$(10^2,10^{-1})$ vs $(10^5,10^{-4})$		*18.3%*	*3.0%*
7	$[10^2]$ vs $(10^6,10^{-4})$	$(10^2,10^{-1})$ vs $(10^6,10^{-5})$		*20.3%*	*2.5%*
8	$(10^3,10^{-1})$ vs $(10^4,10^{-2})$	$(10^3,10^{-2})$ vs $(10^4,10^{-3})$		*15.0%*	*10.0%*
9	$(10^3,10^{-1})$ vs $(10^5,10^{-3})$	$(10^3,10^{-2})$ vs $(10^5,10^{-4})$		*15.7%*	*5.7%*
10	$(10^3,10^{-1})$ vs $(10^6,10^{-4})$	$(10^3,10^{-2})$ vs $(10^6,10^{-5})$		*15.1%*	*6.0%*
11	$(10^4,10^{-2})$ vs $(10^5,10^{-3})$	$(10^4,10^{-3})$ vs $(10^5,10^{-4})$		*18.5%*	*9.3%*
12	$(10^4,10^{-2})$ vs $(10^6,10^{-4})$	$(10^4,10^{-3})$ vs $(10^6,10^{-5})$		*20.7%*	*6.1%*
13	$(10^5,10^{-3})$ vs $(10^6,10^{-4})$	$(10^5,10^{-4})$ vs $(10^6,10^{-5})$		*13.6%*	*7.9%*
14	$(10^3,10^{-1})$ vs $(10^5,10^{-3})$		$(10^3,10^{-3})$ vs $(10^5,10^{-5})$	*27.4%*	*4.7%*

Note. The first column numbers the common-ratio Allais cases. The next three columns present the comparison under the respective expected payoffs of EV = 1, EV =10, and EV = 100. The last two columns display rates of Allais behavior and the corresponding rates of reverse Allais behavior. The fourteen common-ratio comparisons are grouped in terms of three instances of expected payoffs of *10* versus *1* (Items 1 – 3), six instances of expected payoffs of *100* versus *10* (Items 4 – 13) and one instance of expected payoffs of *100* versus *1* (Item 14). The comparisons are further arranged with those involving a sure outcome appearing first.

We further test for the possible presence of a certainty effect in the observed Allais behavior (Kahneman and Tversky, 1979). Examining common-ratio comparisons involving a sure outcome (Items 1, 2, 4 – 7) with those without a sure outcome in the rest of the comparisons (Items 3, 8 – 14), we find that the average proportion of common-ratio Allais behavior is *18.8* percent for the former. This is not significantly different from *16.7* percent for the latter (D =

1.118, p > 0.1). In view of the predominantly risk averse nature of the observed choice behavior at expected payoff of *100*, we test separately for certainty effect for the four comparisons (Items 4 – 7) involving *100* as a sure outcome compared the six comparisons (Items 8 – 13) without involving sure outcomes across the same expected payoffs of *100* and *10*. Here, we find significant difference ($D = 1.93$ $p < 0.03$) between the average incidence of Allais behavior of *20.1* percent in the former compared to *16.4* percent for the latter suggesting the incidence of a certainty effect. As anticipated, focusing on the two comparisons (Items 1 and 2) involving a sure outcome of *10* and the corresponding comparison in Item 3 with the same expected payoffs of *10* and *1* does not yield a certainty effect ($D = -1.19$, $p > 0.1$).

Summarizing, we have the following overall observation.

Observation 2B. *Subjects exhibit systematic equal-mean common-ratio Allais behavior for small probabilities. We observe a certainty effect for common-ratio comparisons involving sure payoffs of 100, but not for sure payoffs of 10.*

6. Discussion and Conclusion

Our main experimental findings add to two stylized observations in the literature—people tend to be risk seeking when facing longshot risks, and among such people, there is a further tendency to favor bets with smaller winning probabilities for the same expected payoff. We find a stake-size dependence of longshot preference with stronger incidence of risk proneness for lotteries with expected payoffs of *1* and *10* and predominant risk aversion at expected payoff of *100*. Rather than monotonic longshot preference, subjects who are risk seeking tend to favor an intermediate winning probability more than going for the smallest possible winning probability. Interestingly, the favored winning probability itself increases with expected payoff.

Our findings are underpinned by an overall tendency for subjects to switch from being risk seeking at low expected payoff to being risk averse at high expected payoff. We observe this in three ways: (i) when the winning probability is fixed, (ii) when the winning outcome is fixed, and (iii) when the winning outcomes in pairs of lotteries are is fixed, leading to the same ratio of winning probabilities. This latter tendency underpins the common ratio Allais paradox which has been often cited as evidence of a certainty effect. In comparing the incidence of Allais behavior with and without a certainty payoff, we find a greater incidence of certainty effect for comparisons involving sure payoffs of 100 but not for sure payoffs of 10, revealing a dependence on stake size.

The findings from our experiment help discriminate among possible specifications of the different utility models in choice under risk. For EU, we find that Markowitz's S-shape utility function performs considerably better than the Friedman-Savage reverse S-shape utility function which includes the widely used three-moment EU model incorporating skewness preference. Using a concave utility function with decreasing elasticity, both RDU and WU can account for the main observations of our experiment through overweighting of winning probabilities. RDU overweights directly with a probability weighting function which is initially concave. With an increasing weight function defined on outcomes, the higher the winning outcome, the more WU overweights the winning probability. In the limiting case of a power utility function with constant elasticity, these two models differ in their implications. Here, RDU is compatible with outcome scale aversion but not with probability scale aversion. While RDU can exhibit single-peak longshot preference, the favored winning odds being fixed does not accord well with the migratory pattern. By contrast, WU can exhibit a fuller range of longshot preference behavior; however, WU is not compatible with outcome scale aversion when the weight function is increasing.

Intransitive choice and violation of stochastic dominance. We relate the observed intransitive choices to violation of stochastic dominance in terms of four binary choices in which one lottery stochastically dominates another. As observed in the preceding section, between *12* and *15* percent of subjects exhibit intransitive choice at various levels of expected payoffs. Note that each of these are far lower than the corresponding chance rate of *63* percent. In terms of stochastic dominance, *50* percent of our subjects show no violation, *31* percent have *one* violation, *15* percent have *two* violations, *4* percent have *three* violations, and the remaining *1* percent have *four* violations. Appendix C discusses the robustness of these results with respect to violation of stochastic dominance. We use the observed degree of dominance violation—which may reflect subjects' level of attention and effort in participation—to assess the extent to which dominance violation may be related to intransitive choice. We find that subjects with dominance violation are more likely to exhibit intransitive choice ($p < 0.001$). More specifically, among subjects without violation of first order stochastic dominance, *31* percent exhibit intransitive choice. In contrast, among subjects with some violation of first order stochastic dominance, *45* percent exhibit intransitive choice.

Accounting for intransitive choice. The association between violation of stochastic dominance and preference intransitivity suggests that the observed intransitive choice may be linked to inattention or lack of effort in participating in our experiment. Intransitive choice may arise from the decision maker having a random component in his utility function (Marschak, 1960), which suggests an extension of the transitive choice models considered by adding a possibly random utility component. A more recent perspective, traceable to Machina (1985), is to posit that stochastic choice can arise from the decision maker having an intrinsic preference for randomizing—a form of preference convexity—especially among options that are otherwise proximate in terms of preference. This latter behavior is compatible with RDU with a π function that is initially concave, but not with betweenness-based models including WU and DAU that rule out such randomization preference. The present experimental setup does not encompass a test of the degree to which different stochastic choice models may account for the observed intransitive choice.

In their nontransitive model of salience theory, Bordalo, Gennaioli, and Shleifer (2012) incorporate the idea of salience of possible states arising from binary choices. The model overweights the probabilities of outcomes associated with the more salient states. In doing so, this model can account for the phenomenon of preference reversal along with Allais type behavior. In our setting of single-prize lotteries, we demonstrate in Appendix B that salience theory delivers the predominance transitive choice rather than intransitive choice which is far less frequent than chance rate. In particular, for longshot risks, we show that salience theory behaves similarly as RDU, making use of the same probability weighting function as Gul (1991). Consequently, salience theory will exhibit similar longshot related properties in our experimental setting.

Longshot preference in market settings. Our results shed light on the understanding of gambling behavior and the working of gaming markets. To account for the phenomenon of favorite-longshot bias in different market settings (see Ottaviani and Sørensen, 2008 for a review), researchers have offered theoretical accounts based on asymmetric information (Ali, 1977; Shin, 1991); heterogeneity in bettors' behavior (Hurley and McDonough, 1995; Ottaviani and Sørensen 2006), misperception of winning probability and probability weighting (Griffith, 1949; Jullien and Salanié, 2000; Snowberg and Wolfers, 2010), and bettors being generally risk seeking (Quandt, 1996; Golec and Tamarkin, 1998). Barseghyan, Molinari, and O'Donoghue, (2013) have noted that the estimation of winning odds relies on aggregate data, and cannot differentiate between estimating the winning probability versus weighting the winning probability. In our experimental setting, using fixed-odds-fixed-outcome lotteries enables a focus on nonlinear probability weighting rather than an estimation of probability in accounting for the observed choice behavior.

Our findings of a greater incidence of single-peak longshot preference over monotonic longshot preference suggests a limit to the favorite-longshot bias phenomenon. This may conceivably be reflected in the design of racetrack betting in terms of the number of horses in a typical race and the range of winning odds available. The presence of 1D, 3D, and 5D lottery tickets in China also reveals heterogeneity in the demand for lottery products in terms of favorite winning probabilities. Interestingly, based on a report on sales of lottery products in China from Caitong Consultancy, a lottery research institute of sina.com, the 2015 annual sales for 3D lottery tickets is RMB20.5 billion of compared to RMB3.2 billion for 5D lottery tickets (http://caitong.sina.com.cn/n/ob/2016-01-21/doc-ifxnrahr8607137.shtml), suggesting that many lottery purchasers do not favour 10^{-5}. Relatedly, the strong incidence of longshot preference at expected payoffs of *1* and *10* but not *100* corroborates a general observation about lottery ticket pricing. Lottery tickets tend to be priced low; scaling up the price together with their prizes may not pay.

In the insurance market, it is commonly suggested that the demand for insurance could be driven by the overweighting of small probabilities of large losses. Kunreuther and Pauly (2003) argue that people often ignore extreme small probabilities, leading to underpurchasing of insurance for disasters. Relatedly, McClelland, Schulze, and Coursey (1993) propose that some people fully ignore small probabilities while other people overweight them. The literature further suggests that people tend to be more pessimistic as stakes increase (see, e.g., Etchart-Vincent, 2004; Barseghyan, Molinari, and O'Donoghue, 2013). It is natural to ask whether we may observe the counterparts in the insurance setting what we observe for longshot gain-oriented risks. Would we find single-peak preference in insuring longshot hazards? Is there a stake-size effect in attitude towards longshot hazards? Further research towards addressing these questions may help shed light on the design of insurance policies as well as complement the formulation of corporate strategy for risk management.

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Appendix A. Supplementary Tables

Table A1. Lotteries used in the experiment.

Outcome	EV = 1		EV = 10		EV = 100	
x	p	Lottery	p	Lottery	p	Lottery
1	*1*	Cash	-	-	-	-
10	*10^{-1}*	1D	*1*	Cash	-	-
10^2	-	-	*10^{-1}*	Same 1D	*1*	Cash
10^3	*10^{-3}*	3D	*10^{-2}*	Different 3D	*10^{-1}*	Different 3D
2×10^3	-	-	*5×10^{-3}*	*5* same 3D	-	-
5×10^3	-	-	*2×10^{-3}*	*2* same 3D	-	-
10^4	-	-	*10^{-3}*	Same 3D	*10^{-2}*	*10 x 10* 3D
2×10^4	-	-	-	-	*5×10^{-3}*	*50* same 3D
5×10^4	-	-	-	-	*2×10^{-3}*	*20* same 3D
10^5	*10^{-5}*	5D	*10^{-4}*	Different 5D	*10^{-3}*	Same 3D
2×10^5	-	-	*5×10^{-5}*	*5* same 5D	-	-
5×10^5	-	-	*2×10^{-5}*	*2* same 5D	-	-
10^6	-	-	*10^{-5}*	Same 5D	*10^{-4}*	*10 x 10* 5D
2×10^6	-	-	-	-	*5×10^{-5}*	*50* same 5D
5×10^6	-	-	-	-	*2×10^{-5}*	*20* same 5D
10^7	-	-	-	-	*10^{-5}*	Same 5D

Note. Besides the lotteries listed in Figure 1, this table lists an additional eight lotteries involving winning probabilities of 2×10^{-3}, 5×10^{-3}, 2×10^{-5}, and 5×10^{-5}. The choice frequencies of the additional lotteries under the three levels of expected payoffs – EV = 1, EV =2, and EV =3 – appear similar to those corresponding to the adjacent lotteries. Column 1 displays the winning outcome. Columns 2, 4, and 6 display the winning probabilities under the three expected payoffs respectively. Columns 3, 5, and 7 display our implementation of the individual lotteries using different combinations of 1D, 3D, and 5D tickets for the three expected payoffs respectively.

Table A2. Comparisons involving stochastic dominance

1A	1/100,000 chance receiving RMB100,000 and 99,999/100,000 chance of receiving 0.
1B	1/100,000 chance receiving RMB10,000 and 99,999/100,000 chance of receiving 0.
2A	1/10,000 chance receiving RMB100,000 and 9,999/10,000 chance of receiving 0.
2B	1/100,000 chance receiving RMB100,000 and 99,999/100,000 chance of receiving 0
3A	50/1,000 chance receiving RMB980,500/1,000 chance of receiving RMB98,and 450/1,000 chance of receiving 0
3B	50/1,000 chance receiving RMB9,800,500/1,000 chance of receiving RMB980,and 450/1,000 chance of receiving 0
4A	10/100,000 chance receiving RMB1,000,000, 5000/100,000 chance of receiving RMB1,000,and 94,990/100,000 chance of receiving 0.
4B	5/100,000 chance receiving RMB1,000,000, 5000/100,000 chance of receiving RMB1,000,and 94995/100,000 chance of receiving 0 Yuan

Note. The table presents four pairs of lotteries in which one option dominates the other option in terms of first order stochastic dominance.

Table A3. Frequencies of the twenty four transitive choice patterns

Small-probability risk attitude	Transitive choice patterns				Favored winning probability	Frequencies in %		
						EV = 1	EV = 10	EV = 100
SP over $(0, 10^{-1}]$	0	5	3	1	10^{-1}	*8.7****	*9.7****	*1.70*
	5	0	3	1	10^{-1}	*1.40*	*3.8****	*0.60*
	5	3	0	1	10^{-1}	*1.10*	*5.4****	*5.3****
	0	1	5	3	10^{-3}	*21.1****	*7.8****	*0.50*
	0	5	1	3	10^{-3}	*3.5****	*2.9****	*0.60*
	5	0	1	3	10^{-3}	*0.40*	*0.40*	*0.10*
Monotonic	0	1	3	5	10^{-5}	*23.6****	*11.6****	*3.2****
SP over $(0, 10^{-3}]$	1	0	5	3	10^{-3}	*2.6***	*0.008*	*0.50*
	1	5	0	3	10^{-3}	*0.10*	*0.20*	*0.00*
	5	1	0	3	10^{-3}	*0.10*	*0.00*	*0.00*
	1	0	3	5	10^{-5}	*2.8****	*0.014*	*0.70*
SP over $(0, 10^{-5}]$	3	1	0	5	10^{-5}	*0.00*	*0.00*	*0.10*
	1	3	0	5	10^{-5}	*0.20*	*0.00*	*0.50*
Purely Risk Averse	5	3	1	0	1	*8.6****	*20.3****	*47.2****
	3	5	1	0	1	*1.30*	*5.7****	*14.0****
	5	1	3	0	1	*0.50*	*1.00*	*2.8****
	1	5	3	0	1	*0.50*	*0.00*	*0.50*
	1	3	5	0	1	*1.20*	*2.20*	*2.8****
	3	1	5	0	1	*0.20*	*0.50*	*1.00*
Others	0	3	5	1	-	*3.0****	*4.2****	*0.60*
	3	5	0	1	-	*0.70*	*2.3**	*1.70*
	3	0	5	1	-	*0.60*	*1.00*	*0.50*
	0	3	1	5	-	*2.3**	*0.016*	*0.80*
	3	0	1	5	-	*0.20*	*0.20*	*0.00*
					Total	*84.4****	*82.9****	*86.7****

Note. The table presents the frequencies of the twenty four transitive choice patterns. They include thirteen single-peak (SP) choice patterns, six purely risk averse choice patterns, and five choice patterns under "Others". Each choice pattern lists the four options in ascending order of preference, e.g., 1053 means that $(10^3, 10^{-3}) \succ (10^5, 10^{-5}) \succ [1] \succ (10, 10^{-1})$. Against a chance rate of *1.6* percent (*1* out of *64* possible patterns), the per cell threshold frequencies for the three levels of significance *10%, **5%, ***1% are 2.3 percent, 2.5 percent, and 2.8 percent, respectively.

Table A4. Frequencies of longshot preferring and risk averse choice patterns

Small-Probability Risk Attitude	Chance Rate	Frequency (%)		
		EV = 1	EV = 10	EV = 100
Single-peak over $(0, 10^{-1}]$	7/64	*59.5****	*41.6****	*12.0*
Single-Peak@10^{-1}	3/64	*11.2****	*19.0****	*7.6****
Single-Peak@10^{-3}	3/64	*25.0****	*11.0****	*1.2*
Single-Peak@10^{-5}	1/64	*23.3****	*11.6****	*3.2***
Single-peak over $(0, 10^{-3}]$	4/64	*5.7*	*2.5*	*1.2*
Single-Peak@10^{-3}	3/64	*2.9*	*1.1*	*0.5*
Single-Peak@10^{-5}	1/64	*2.8**	*1.4*	*0.7*
Single-peak over $(0, 10^{-5}]$	2/64	*0.2*	*0.0*	*0.6*
Purely risk averse	6/64	*12.3****	*29.7****	*68.3****
Others	5/64	*6.8*	*9.2*	*3.6*
Total Transitive	24/64	*84.6*	*82.9*	*86.7*

Note. The table summarizes the frequencies of single-peak and purely risk averse choice patterns in Table A4 along with "Others" under the three levels of expected payoffs (EV =1, EV =2, and EV = 100). We test whether the observed frequency is significantly higher than the chance rate at three levels of significance—10 percent, 5 percent, and 1 percent—indicated by *, **, and *** using the proportion tests.

Table A5. Migration of favoured winning probabilities across expected payoffs

		Panel A: Expected payoffs 1 and 10				
		Expected payoff 10				
		1	10^{-1}	10^{-3}	10^{-5}	Total
Expected payoff 1	1	*95*	*4*	*0*	*0*	*99*
	10^{-1}	*57*	*26*	*2*	*1*	*86*
	10^{-3}	*18*	*79*	*53*	*15*	*165*
	10^{-5}	*4*	*14*	*37*	*86*	*141*
	Total	*174*	*123*	*92*	*102*	*491*

		Panel B: Expected payoffs 10 and 100				
		Expected payoff 100				
		1	*10^{-1}*	*10^{-3}*	*10^{-5}*	*Total*
Expected payoff 10	1	*231*	*4*	*0*	*2*	*237*
	10^{-1}	*127*	*20*	*1*	*0*	*148*
	10^{-3}	*43*	*20*	*7*	*4*	*74*
	10^{-5}	*29*	*4*	*3*	*29*	*65*
	Total	*430*	*48*	*11*	*35*	*524*

		Panel C: Expected payoffs 1 and 100				
		Expected payoff 10				
		1	10^{-1}	10^{-3}	10^{-5}	Total
Expected payoff 1	1	*96*	*0*	*0*	*1*	*97*
	10^{-1}	*83*	*7*	*0*	*0*	*90*
	10^{-3}	*161*	*34*	*5*	*2*	*202*
	10^{-5}	*96*	*12*	*7*	*32*	*147*
	Total	*436*	*53*	*12*	*35*	*536*

Note. Panel A shows the counts for subjects who migrate from being either risk averse, denoted by "*1*", or having single-peak longshot preference at 10^{-1}, 10^{-3}, or 10^{-5} at expected payoff of *1* to being either risk averse, or having single-peak longshot preference at 10^{-1}, 10^{-3}, or 10^{-5} at expected payoff of *10*. Panel B (resp: Panel C) shows the corresponding migration counts for subjects from expected payoff of *10* (resp: *1*) to expected payoff of *100*. Pearson's chi-squared tests are highly significant ($p < 0.001$).

Table A6. Proportion (%) of scale-averse behavior as expected payoff increases

Panel A: Fixed winning probability					
Probability	SSS	SSA	SAA	AAA	Others
10^{-1}	*16.9*	*37.3*	*20.6*	*17.8*	*7.4*
10^{-3}	*14.8*	*37.9*	*25.7*	*18.1*	*2.3*
10^{-5}	*13.6*	*37.2*	*28.3*	*19.0*	*1.8*

Panel B: Fixed winning outcome					
Outcome	SSS	SSA	SAA	AAA	Others
10^{3}	*16.9*	*37.1*	*24.4*	*18.9*	*3.8*
10^{5}	*14.1*	*37.6*	*26.8*	*18.9*	*2.6*

Note. Panel A presents the percentage of choice patterns across the three levels of expected payoffs keeping the winning probability constant. Panel B presents the percentage of choice patterns across the three levels of expected payoffs keeping the winning outcome constant. SSS (risk seeking at all three levels of expected payoffs), SSA (risk seeking at expected payoffs of *1* and *10*, and risk averse at expected payoffs of *100*), SAA (risk seeking at expected payoffs of *1*, and risk averse at expected payoffs of *10* and *100*), and AAA (risk averse at all three levels of expected payoffs).

Appendix B. Salience theory and transitive choice

Bordalo, Gennaioli, and Shleifer (2012) propose a salience theory model, in which the decision maker overweights the probability of a more salient state. We first discuss how salience theory evaluates (y, p) in relation to $[py]$ which yields a winning state (y, py) and a losing state $(0, py)$. The decision maker assigns different salience values to these two states using a salience function σ. One salience function $\sigma(x, y)$ taking the form $\frac{|x-y|}{x+y+\theta}$ delivers higher salience at (y, py) than at $(0, py)$ when $\theta/y > p^2/(1-2p)$. When the salience of the winning state (y, py) exceeds that of the losing state $(0, py)$, the decision maker overweights the winning outcome using a probability weighting of $p/(p + (1-p)\delta)$ with $\delta < 1$. Otherwise, the winning state is underweighted by $p\delta/(p\delta + 1 - p)$. Notably, the probability weighting given by $p/(p + (1-p)\delta)$ coincides with that of DAU with $\delta = 1 + \beta$. It follows that salience theory can account for Observation 1 when u has decreasing elasticity. DAU can also exhibit both outcome and probability scale aversion since its probability weighting function switches from the winning state (y, py) being more salient to the losing state $(0, py)$ being more salient as the winning probability p or the winning outcome y increases.

We next show that salience theory always exhibits transitivity for binary choices among three statistically independent equal-mean lotteries $(x, m/x)$, $(y, m/y)$, and $(z, m/z)$ with $x < y < z$. Following Bordalo, Gennaioli, and Shleifer (2012), we adopt a linear u function. For a choice between $(x, m/x)$ and $(y, m/y)$, there are four states: $(0, 0)$, (x, y), $(0, x)$, and $(0, y)$. With the condition on ordering imposed in the model, the salience ranking is lowest for $\sigma(0, 0)$, highest for $\sigma(0, y)$, and undetermined between $\sigma(x, y)$ and $\sigma(0, x)$. If $\sigma(x, y) > \sigma(0, x)$, $(y, p_2) \succcurlyeq (x, p_1)$ as the state associated with y is always overweighted. If $\sigma(0, x) > \sigma(x, y)$, we compare the utility difference between $(y, m/y)$ and $(x, m/y)$ given by

$$\frac{m}{y}\left(1-\frac{m}{x}\right)y - \delta\left(1-\frac{m}{y}\right)\frac{m}{x}x + \delta^2\frac{m}{y}\frac{m}{x}(y-x) = (1-\delta)[1-\frac{m}{x} - \delta\left(\frac{m}{x}-\frac{m}{y}\right)].$$

This utility difference exceeds 0 if m/x exceeds 1/2. It follows that $(y, m/y) \succcurlyeq (x, m/x)$ regardless of the salience between $\sigma(x, y)$ and $\sigma(0, x)$. Similarly, we have $(z, m/z) \succcurlyeq (y, m/y)$ and $(z, m/z) \succcurlyeq (x, m/x)$. That is, salience theory with linear u exhibits transitive choice among the statistically independent lotteries in our experiment.

Appendix C. Robustness Checks

This appendix examines the robustness of the results with respect to violation of first order stochastic dominance. We make use of the observed degree of violation, which may reflect the subjects' level of attentiveness and effort in participating in our experiment, to test whether this factor influences the observed choice behavior. Subjects are divided into two groups—one without violations and another with at least one violation. For those without violations of dominance, the proportion of transitive patterns is between 87.1 percent and 91.9 percent, which is significantly higher than those with violations of dominance (between 78.5 percent and 81.8 percent) ($p < 0.001$). The proportion of longshot preference at (0, 10-1] is similar for those with violations of dominance and those without violations of dominance (58.4 percent versus 60.5 percent for expected payoffs of *1*; 40.7 percent versus 42.3 percent for expected payoffs of *10*; 14.4 percent versus 9.6 percent for expected payoffs of *100*). Overall, those without violations of dominance are more risk averse than those with violations (*9.3* percent versus 15.3 percent for expected payoffs of *1*; 24.6 percent versus *34.7* percent for expected payoffs of *10*; 60.5 percent versus 78.2 percent for expected payoffs of *100*).

For scale aversion, the proportion of risk aversion across the three levels of expected payoffs for those without violations of dominance is between 21 percent and 23 percent for either fixed winning probability or fixed winning outcome, which is significantly larger than those with violations of dominance (about 15 percent) ($p < 0.001$), while the proportion of risk seeking across the three levels of expected payoffs for those without violations of dominance is between 10 percent and 15 percent, which is significantly smaller than those with violations of dominance (between 18 percent and 21 percent) ($p < 0.001$). The overall proportions of Allais behavior for these two groups appear similar. While subjects without violations exhibit a significantly lower incidence of Allais behavior for expected payoffs across *10* and *100* relative to those with violations ($p < 0.001$), we do not observe a significant difference in the incidence of Allais behavior for expected payoffs across *1* and *10* ($p > 0.584$), and for expected payoffs across *1* and *100* ($p > 0.104$). We report the details in Tables C1, C2, and C3. In sum, while there are some observed differences for subjects who are more prone to violations of stochastic dominance, the qualitative features of the observed longshot preference behavior, the switching behavior in risk attitudes across expected payoffs, and the equal-mean common-ratio Allais behavior remain robust.

Table C1. Frequencies of single-peak and risk averse choice patterns

Small-Probability Risk Attitude	Chance Rate	With Violation (%)			Without Violation (%)		
		EV = 1	EV = 10	EV = 100	EV = 1	EV = 10	EV = 100
Single-peak over (0, 10^{-1}]	7/64	*58.4*	*40.7*	*14.4*	*60.5*	*42.3*	*9.6*
Single-Peak@10^{-1}	3/64	*9.6*	*14.1*	*8.3*	*12.9*	*23.7*	*6.7*
Single-Peak@10^{-3}	3/64	*21.5*	*11.5*	*1.4*	*28.2*	*10.5*	*0.9*
Single-Peak@10^{-5}	1/64	*27.2*	*15.1*	*4.5*	*19.4*	*8.1*	*1.9*
Single-peak over (0, 10^{-3}]	4/64	*6.5*	*2.9*	*1.4*	*4.8*	*2.1*	*0.9*
Single-Peak@10^{-3}	3/64	*2.6*	*1.4*	*0.7*	*3.1*	*0.7*	*0.2*
Single-Peak@10^{-5}	1/64	*3.8*	*1.4*	*0.7*	*1.7*	*1.4*	*0.7*
Single-peak over (0, 10^{-5}]	2/64	*0.2*	*0.0*	*1.0*	*0.2*	*0.0*	*0.2*
Purely risk averse	6/64	*9.3*	*24.6*	*60.5*	*15.3*	*34.7*	*78.2*
Others	5/64	*7.4*	*10.3*	*4.3*	*6.2*	*8.1*	*2.8*
Total Transitive	24/64	*81.8*	*78.5*	*81.5*	*87.1*	*87.3*	*91.8*

Note. The table summarizes the frequencies of single-peak and purely risk averse choice patterns in Table A4 along with "Others" for those with and without violations of first-order stochastic dominance.

Table C2. Effect of violation of stochastic dominance on choice across expected payoffs

		Panel A: Fixed winning probability			
Probability	Choice Pattern	With Violation		Without Violation	
		Mean	Std. Dev.	Mean	Std. Dev.
10^{-1}	AAA	*15.6*	*1.8*	*20.1*	*2.0*
	SAA	*19.4*	*1.9*	*21.8*	*2.0*
	SSA	*37.1*	*2.4*	*37.6*	*2.4*
	SSS	*19.4*	*1.9*	*14.4*	*1.7*
	Others	*8.6*	*1.4*	*6.2*	*1.2*
10^{-3}	AAA	*14.8*	*1.7*	*21.3*	*2.0*
	SAA	*25.1*	*2.1*	*26.3*	*2.2*
	SSA	*37.3*	*2.4*	*40.9*	*2.4*
	SSS	*18.9*	*1.9*	*10.8*	*1.5*
	Others	*3.8*	*0.9*	*0.7*	*0.4*
10^{-5}	AAA	*15.1*	*1.8*	*23.0*	*2.1*
	SAA	*25.8*	*2.1*	*30.9*	*2.3*
	SSA	*37.6*	*2.4*	*36.8*	*2.4*
	SSS	*18.7*	*1.9*	*8.6*	*1.4*
	Others	*2.9*	*0.8*	*0.7*	*0.4*
		Panel B: Fixed winning outcome			
Outcome	Choice Pattern	With Violation		Without Violation	
		Mean	Std. Dev.	Mean	Std. Dev.
1,000	AAA	*15.6*	*1.8*	*20.6*	*2.0*
	SAA	*23.4*	*2.1*	*25.4*	*2.1*
	SSA	*36.6*	*2.4*	*37.6*	*2.4*
	SSS	*20.6*	*2.0*	*13.2*	*1.7*
	Others	*3.8*	*0.9*	*3.3*	*0.9*
100,000	AAA	*15.6*	*1.8*	*22.2*	*2.0*
	SAA	*24.9*	*2.1*	*28.7*	*2.2*
	SSA	*38.0*	*2.4*	*37.1*	*2.4*
	SSS	*18.2*	*1.9*	*10.0*	*1.5*
	Others	*3.3*	*0. 9*	*1.9*	*0.7*

Note. Panel A presents the mean and standard deviation of the incidence (in %) of choice patterns across the three levels of EVs keeping the winning probability constant for those with and without violations of first-order stochastic dominance. Panel B presents the mean and standard deviation (in %) of the incidenceof choice patterns across the three levels of EVs keeping the winning outcome constant for those with and without violations. SSS (risk seeking at all three EVs), SSA (risk seeking at EV = 1 and EV = 10, and risk averse at EV = 100), SAA (risk seeking at EV = 1, and risk averse at EV = 10 and EV = 100), and AAA (risk averse at all three EVs). Using multinomial logistic regression, we find that no-violation subjects tend to exhibit more AAA and less SSS, suggesting that they are generally more risk averse (10^{-1}, $p < 0.103$; 10^{-3}, $p < 0.001$; 10^{-5}, $p < 0.001$; 10^{3}, $p < 0.039$; 10^{5}, $p < 0.002$). The overall single-switch behaviors for both groups appear similar.

Table C3. Effect of violation of stochastic dominance on Allais behavior

Allais Behavior	With violation		Without violation	
	Mean	Std. Dev.	Mean	Std. Dev.
EV = 1 vs. EV =10	*17.5*	*24.1*	*16.3*	*23.5*
EV = 10 vs. EV = 100	*21.0*	*17.8*	*14.9*	*16.6*
EV = 1 vs. EV = 100	*29.9*	*45.8*	*249*	*43.3*

Note. The table lists the mean and standard deviation (in %) of the incidence of Allais behavior for those with violations (columns 2 and 3), those with violations (columns 4 and 5). The overall proportions of Allais behavior for these two groups appear similar. While subjects without violations exhibit a significantly lower incidence of Allais behavior for EV = 10 vs EV = 100 than those with violations, we do not observe a significant difference between EV = 1 and EV = 10, and between EV = 1 and EV = 100.

Appendix D: Experimental Instructions

Thank you for participating in this study on decision making. Please read the following instructions carefully before you make any decisions.

Tasks: In this study, you will make a number of binary choices as illustrated in the following two examples.

Example 1. Which of the following two options will you choose?

A. 1/1000 chance of receiving 2000 Yuan, 999/1000 chance of receiving 0 Yuan

B. 1/100,000 chance of receiving 200,000 Yuan, 99,000/100,00 of receiving 0 Yuan

Choosing A means that you have 1/1000 chance of receiving 2000 Yuan and 999/1000 chance of receiving 0 Yuan. Choosing B means that you have 1/100,000 chance of receiving 200,000 Yuan and 99,000/100,000 chance of receiving 0 Yuan.

Example 2. Which of the following two options will you choose?

A. 1/1000 chance of receiving 100,000 Yuan, 999/1000 chance of receiving 0 Yuan

B. receiving 100 Yuan for sure

Choosing A means that you have 1/1000 chance of receiving 100,000 Yuan and 999/1000 chance of receiving 0 Yuan. Choosing B means that you receive 100 Yuan for sure.

In each round, you choose between two options, and there are 100 rounds in total. The probability and amount of money may be different in each round. We will use lotteries with different combinations of probability and amount of money.

Details of Rules for the Lottery: Three types of lotteries are used in this study, i.e., “Array 3” and “Array 5” of China Sports Lottery, and “3D” of China Welfare Lottery. You may refer to the detailed rules of these three lotteries. Below is a brief introduction of these three types of lotteries.

Array 3. Buyers can choose a three-digit number from 000 to 999. If the number chosen by the buyer is the winning number (same digits in the same order), the buyer of the lottery wins 1000 Yuan. That is, the probability of winning 1000 Yuan is 1/1000 for a randomly chosen number. For example, if the winning number is 543 and you have ten lotteries with the number 543, you will receive 10,000 Yuan. That is, you have 1/1000 chance to win 10,000 Yuan with ten lotteries of the same number.

Array 5. Buyers can choose a five-digit number from 00000 to 99999. If the number chosen by the buyer is the winning number (same digits in the same order), the buyer of the lottery wins 100,000 Yuan. That is, the probability of winning 100,000 Yuan is 1/100,000 for a randomly chosen number.

For example, if the winning number is 54321 and you have ten lotteries with the number 54321, you will receive 1000,000 Yuan. That is, you have 1/100,000 chance to win 1000,000 Yuan with ten lotteries of the same number.

3D. 3D is similar to Array 3. Buyers can bet on a three-digit number from 000 to 999. If the number chosen by the buyer is the winning number (same digits in the same order), the buyer of the lottery wins 1000 Yuan. That is, the probability of winning 1000 Yuan is 1/1000 for a randomly chosen number. Lottery 3D has another two ways of betting.

"2D" Betting: Buyers can bet on the first two digits, last two digits, or the first and last digit of a three-digit number from 000 to 999. The chosen two digits should have the same order and be in the same position as the winning number. The winning amount is 98 Yuan for each ticket.

"1D" Betting: Buyers can bet on the ones, tens, and hundreds of a three-digit number from 000 to 999. The chosen digit should have the same order and be in the same position as the winning number. The winning amount is 10 Yuan for each ticket.

Detailed rules for "Array 3" in China Sports Lottery:

http://www.lottery.gov.cn/news/10006630.shtml

Detailed rules for "Array 5" in China Sports Lottery:

http://www.lottery.gov.cn/news/10006657.shtml

Detailed rules for "3D" in China Welfare Lottery:

http://www.bwlc.net/help/3d.jsp

We will implement the corresponding probability and the amount by combining different types of lotteries. In Option A of Example 1, you have 1/1000 chance of receiving 2000 Yuan with two "Array 3" lotteries with the same number. In Option A of Example 2, you have 1/1000 chance of receiving 100,000 Yuan with 100 "Array 3" with the same number.

In a similar manner, you will get lottery combinations with different probabilities of winning various amounts. In this experiment, all the numbers of the lotteries are generated randomly by a computer. We will buy these lotteries from lottery stores.

Payment: Every participant in the experiment will get 20 Yuan as a base payment. You have a ten percent chance of receiving an additional payment, which is randomly chosen in the following way. We will add your birthday (year, month, and date—eight numbers in total) to get a one-digit number (0-9). If this number is the same as the sum of the "3D" Welfare lottery on Feb 28, 2013, you will receive the additional payment. That is, you have approximately a ten percent chance of receiving an additional payment.

The amount of the additional payment is decided in the following way. You will be asked to randomly choose one number between 1 and 100, which determines one decision out of your

100 decisions. Your payment will depend on the decision you made on that particular round. If your choice on that round is a certain amount of money, you will get that amount of money. If your choice on that round is a lottery, then your payment is through the lottery.

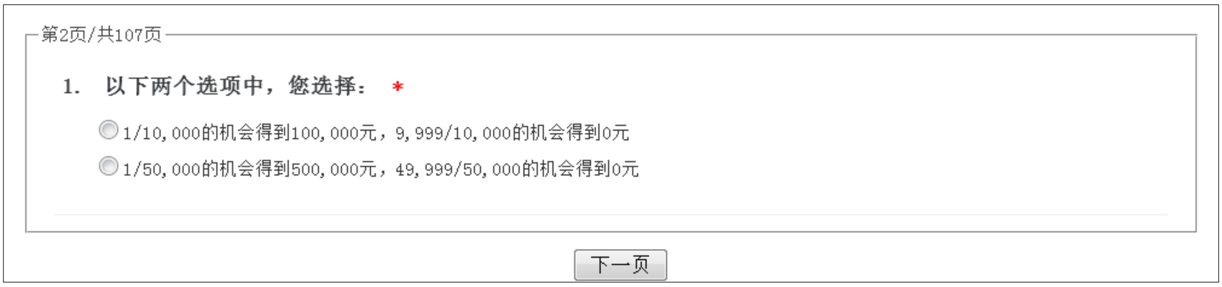
<u>Time for payment</u>: The payment will be implemented around March to April, 2013. The specific date will be announced later.

Summary for Rules:

1. You will be asked to decide between two options in each of the 100 rounds.
2. The probabilities and amounts of money in each decision can be realized by different combinations of lotteries.
3. Each participant will get 20 Yuan as a base payment.
4. Ten percent of participants will be randomly chosen to receive additional payment; the payment will be based on one randomly chosen decision out of the 100 decisions made.

If you have any questions about this experiment, please feel free to email us at b2ess@nus.edu.sg. If you are clear about the instructions, you may start and make your decisions now.

Sample Screen of Choice.



Note. The translation of the sample screen is as follows. Page 2 of 107 Pages: 1. In the following two options, which one will you choose? *1/10,000 chance of receiving 100,000 Yuan, 9,999/10,000 chance of receiving 0; *1/50,000 chance of receiving 500,000 Yuan, 49,999/50,000 receiving 0 Yuan.