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RESEARCH ARTICLE

Probabilistic social preference: how Machina's Mom randomizes her choice

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Abstract We experimentally investigate preference for randomization in social settings, in which the dictator chooses probabilistically between two allocations for herself and an anonymous recipient. We observe substantial proportions of subjects choosing to randomize under various circumstances. The observed patterns have rich implications for various assumptions in social preference models and shed light on recent studies on ex-ante and ex-post social preferences.

Keywords Social preference · Risk preference · Ex-ante fairness · Ex-post fairness

JEL Classification D63 · D64 · C91

1 Introduction

Social preference has been extensively studied both theoretically and experimentally, and many models have been proposed to better describe various aspects of social

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preference, including inequality concern (Fehr and Schmidt 1999; Bolton and Ockenfels 2000 and equality–efficiency trade-off (Andreoni and Miller 2002; Charness and Rabin 2002). When the environment involves risks, it is natural to combine these social preference models with either expected utility or non-expected utility, i.e., decision makers first evaluate the utility of each possible ex-post allocation using their social preferences and then aggregate the utilities based on their risk preferences. The resulting representation is known to capture preference for ex-post fairness since social preference utility is applied for evaluating ex-post allocations. Nevertheless, it has been widely recognized that this *ex-post preference* representation with expected utility cannot account for preference for randomization (Diamond 1967; Machina 1989).[1]

Consider a decision maker facing three options on how to allocate one indivisible token between herself and an anonymous recipient: she could (i) keep the token for herself; (ii) give the token to the recipient; or (iii) keep the token for herself if a coin toss is head and give the token to the recipient otherwise. While the decision maker prefers to keep the token for herself to give the token to the recipient, she may prefer the randomization option as illustrated below.

$$\begin{array}{lcc} & \text{self} & \text{other} \\ \text{Head} & 1 & 0 \\ \text{Tail} & 0 & 1 \end{array} \succ \begin{array}{cc} \text{self} & \text{other} \\ 1 & 0 \end{array} \succ \begin{array}{cc} \text{self} & \text{other} \\ 0 & 1 \end{array}$$

Under ex-post preference with an expected utility specification, the utility of a half/half randomization of the allocations $(1, 0)$ and $(0,\ 1)$ is intermediate between the utilities of the two alternative allocations. Thus, such an ex-post preference is incompatible with the choice behavior above.[2] An intuitive account for the preference ranking above is through *ex-ante preference*, which can be defined as a social preference utility evaluating ex-ante allocation. In the example above, the randomization option delivers an ex-ante expected allocation $(0.5,\ 0.5)$ between the decision maker and the recipient. The decision maker will then exhibit the preference ranking above if she has a preference for ex-ante fairness and prefers $(0.5,\ 0.5)$ to both $(1,\ 0)$ and $(0,\ 1)$.

[1] Machina (1989) considers an example in the context of the social planner's problem, in which a mother is to allocate an indivisible good between two children whom she likes equally well. While the mother is indifferent between allocating the good to either child, she strictly prefers randomizing her choice as in a coin flip. Ex-post preference respecting first-order stochastic dominance implies that any probabilistic allocation has the same valuation as either of the two degenerate allocations. Thus, preference for randomization cannot be explained by ex-post preference respecting dominance.

[2] In this example, the incompatibility between ex-post preference and preference for randomization remains valid for a wider set of utility specifications with the risk aggregator function respecting first-order stochastic dominance. This class of utility functions includes the rank-dependent utility in Quiggin (1982) and those adopting the betweenness approach (Chew 1983; Dekel 1986). Meanwhile, a utility function that is quasiconcave in probabilities, e.g., quadratic utility (Chew et al. 1991), could be compatible with preference for randomization. In fact, the original form of quadratic utility in Chew et al. (1991) still permits first-order stochastic dominance. Ex-post preference with a quadratic utility relaxing dominance is compatible the preference ranking in the example. See Epstein and Segal (1992) for an application of quadratic utility in a social setting, and Machina (1985) for an analysis of stochastic choice in the context of decision making under risk.

A number of theoretical works have been proposed to capture preference for ex-ante fairness. Karni and Safra (2002) axiomatize the individual preference over random allocation procedure of an indivisible good. Trautmann (2009) proposes a model to accommodate preference for process fairness under risk.[3] More recently, a number of studies investigate the implications of ex-ante and ex-post preferences. Fudenberg and Levine (2012) show that preference for ex-ante fairness is incompatible with the independence axiom. They also show that relaxing the independence axiom by replacing expected utility with expected allocation in various social preference specifications would exhibit preference for ex-ante fairness but does not allow for preference for ex-post fairness, and further suggest combining both ex-ante preference and ex-post preference. Saito (2013) provides an axiomatization for a combinational preference with the built-in specification of Fehr and Schmidt (1999).[4] Brock et al. (2013) provide experimental evidence in support of preference with a combination of both ex-ante and ex-post concerns.

This study considers preference for randomization in social situations in which the decision maker (the dictator) chooses whether and how to randomize between various pairs of allocations between herself and an anonymous person (the recipient). More specifically, we consider two possible allocations, (x_1, x_2) and (y_1, y_2), in which x_1 and y_1 represent the payoffs of the dictator, and x_2 and y_2 represent the payoffs of the recipient. Instead of choosing between (x_1, x_2) and (y_1, y_2) with certainty, the dictator chooses probability $p \in [0, 1]$, which delivers allocation (x_1, x_2) with probability p and allocation (y_1, y_2). with probability $1 - p$. The choice situation is denoted as menu $\begin{pmatrix} x_1, & x_2 \\ y_1, & y_2 \end{pmatrix}$ for a pair of allocations (x_1, x_2) and (y_1, y_2). By introducing this probabilistic dictator game, we are able to examine both ex-ante and ex-post preferences in a general social setting. As the two contingent allocations are fixed in our setup, ex-post preference with the risk aggregator function respecting first-order stochastic dominance predicts that the dictator chooses one allocation over the other, i.e., p equals either 0 or 1, instead of choosing interior probability to randomize between the two allocations. In contrast, ex-ante preference allows the dictator to choose interior probabilities $p \in (0, 1)$ to trade-off among self-interest, equality and efficiency concerns. We include 11 choice menus varying the concerns of self-interest, equality and efficiency, as summarized in the allocation triangle in Fig. 1 below.

In choice menus $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$ on the hypotenuse of the triangle in Fig. 1, one allocation permits advantageous inequality for the dictator and the other allocation permits disadvantageous inequality for the dictator with the total pie fixed at 20. Pure selfish concern predicts choosing $p = 1$, while pure ex-ante inequality aversion would induce the decision maker to equally split the chance of winning the larger amount. Choosing interior probabilities $p \in (0, 1)$ would reflect a trade-off between selfish and ex-ante equality concerns.

[3] Trautmann and Wakker (2010) investigate the implications of process fairness and outcome fairness on dynamic consistency.

[4] Other related theoretical studies analyze social preference under risk from the perspective of a social planner; see Fleurbaey (2010) and Chew and Sagi (2012).

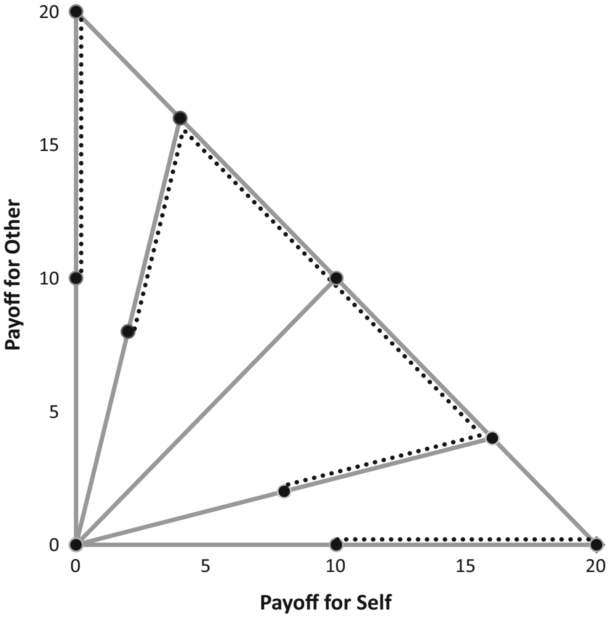

Fig. 1 Experimental menus. In the *triangle*, *one dot* represents a deterministic allocation with the x-axis representing the payoff for the dictator, and the y-axis representing the payoff for the recipient. *One solid/dashed line* connecting *two dots* represents one menu. For instance, the line connecting $(0, 0)$ and (4, 16) represents menu $\begin{pmatrix} 4, & 16 \\ 0, & 0 \end{pmatrix}$

In menus $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$ in the upper part of the triangle in Fig. 1, one allocation has disadvantageous inequality for the dictator, who always gets 0, whereas the other allocation reduces the efficiency either by shrinking the pie to zero or by half for the recipient. Efficiency concern predicts choosing $p = 1$, pure ex-ante inequality aversion predicts choosing $p = 0$, while selfishness predicts indifference between the two allocations. Choosing an interior probability allows the dictator to trade-off between ex-ante equality and efficiency concerns.

Similarly, we consider menus $\begin{pmatrix} 4, & 16 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 4, & 16 \\ 2, & 8 \end{pmatrix}$, which further incorporate selfishness compared with the preceding two menus in the upper part of the triangle and thus enables the trade-off among selfish, equality and efficiency concerns. We include four more menus with advantageous inequality for the dictator as shown in the lower part of the triangle, i.e., $\begin{pmatrix} 20, & 0 \\ 0, & 0 \end{pmatrix}$, $\begin{pmatrix} 16, & 4 \\ 0, & 0 \end{pmatrix}$, $\begin{pmatrix} 20, & 0 \\ 10, & 0 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 8, & 2 \end{pmatrix}$. The last menu, $\begin{pmatrix} 10, & 10 \\ 0, & 0 \end{pmatrix}$, is included as a control since all motives predict choosing $p = 1$.

Overall, we observe a substantial proportion of subjects choosing interior probabilities in various menus when one allocation permits advantageous inequality and the other allocation permits disadvantageous inequality, as well as when one allocation permits disadvantageous inequality and the other allocation shrinks the pie proportionally. By contrast, subjects tend to not randomize when one allocation permits advantageous inequality and the other allocation shrinks the pie proportionally. These patterns reflect a mixture of self-interest, equality and efficiency motives underlying preference for randomization.

Our study contributes to a number of earlier experimental studies related to ex-ante and ex-post social preferences. Karni et al. (2008) test Karni and Safra's (2002) theory with an experiment in which subjects choose among probabilistic allocation of an indivisible good and show that subjects are willing to share the probability of winning the indivisible good. Sandroni et al. (2013) provide an experimental test for the opening example in which subjects choose among three options: allocation with advantageous inequality, allocation with disadvantageous inequality, and randomization between the two allocations. They report that 30% of the subjects choose the randomization option, in support of the prevalence of ex-ante preference. In Bolton and Ockenfels (2010), subjects decide on a number of binary choices between a safe allocation and a risky allocation. They find that subjects are more risk taking when the safe option yields unequal payoffs and that risk taking does not depend on whether the risky option yields unequal payoffs. They further suggest that subjects are more comfortable with ex-post inequality when risky options yield ex-ante equality. In Cappelen et al. (2013), subjects initially make risky decisions and are subsequently asked to make redistribution decisions after the first-stage choices and outcomes are revealed. They find support for ex-ante preference, but the evidence is mixed because ex-post redistribution also takes place. Krawczyk and Le Lec (2010) propose an experiment in which subjects share the probabilities of winning a fixed pie with either independent or dependent draws. They observe that subjects choose to share the probabilities of winning in the dependent draws and that the shared probability in the dependent draws is smaller than the shared probability in the independent draws. Overall, these findings suggest a mix of distributive and procedural fairness preferences. Rohde and Rohde (2011) similarly observe that subjects prefer independent risk to correlated risk across individuals in an experimental setting. Brock et al. (2013) vary the dictator's own risk exposure and the ability to achieve ex-post fairness by allowing the dictator to allocate tokens that could be transformed into lotteries. They argue that neither ex-ante preference nor ex-post preference alone could account for the observations and suggest the need to combine both preferences.

Compared with our design, most of these aforementioned studies stay along the hypotenuse in the allocation triangle, as in our menus $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$, with the size of efficiency fixed.[5] In this study, we explore preference for randomization inside the allocation triangle. Inside the allocation triangle, which incorporates the

[5] Dana et al. (2007) consider cases when subjects are able to leave the relationship between their actions and resulting outcomes uncertain, which gives subjects the moral wiggle room to behave self-interestedly. In the deterministic setup, traditional dictator/ultimatum games examine deterministic social preference along the hypotenuse (see Camerer 2003 for a review) as the subjects choose how to distribute a pie of fixed size. Andreoni et al. (2003) extend the analysis to inside the triangle with a convex ultimatum game, in which the responder can choose to shrink the pie rather than the take-it-or-leave-it option in the standard ultimatum game.

Table 1 Decision table for the subjects

Allocation 1		Allocation 2	
You	Other	You	Other
x_1	x_2	y_1	y_2
p:		***1-p:***	

Given two possible allocations (x_1, x_2) and (y_1, y_2), subjects choose the probability p to implement the first allocation and $1 - p$ to implement the second allocation

additional efficiency motive that is often missing in these recent studies. Moreover, our design enables a systematic examination of the differences between advantageous inequality and disadvantageous inequality under risk, i.e., the lower part of the triangle and upper part of the triangle in Fig. 1. It is difficult to use traditional dictator games to investigate this issue, because the dictators in general choose an allocation in the lower part of the triangle. Overall, our design enables us to examine probabilistic social preference in a variety of situations. Moreover, the observed patterns, as we subsequently show, deliver rich theoretical implications for models on ex-ante and ex-post social preferences.

The rest of this paper is organized as follows. We present the experimental design in Sect. 2. Section 3 discusses the framework of probabilistic social preference theories and their theoretical predictions. We analyze the aggregate patterns and perform an individual-level analysis in Sect. 4. Section 5 concludes.

2 Experimental design

2.1 Experiment I

Subjects are presented with two allocations—Allocation 1 and Allocation 2—between oneself and another anonymous participant who is, randomly selected in the experimental venue (see "Appendix 4" for the experimental instructions). Subjects choose probability p including 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1 to implement Allocation 1, and the complementary probability $1 - p$ to implement Allocation 2 as shown in Table 1 below. Note that probabilities p and $1 - p$ are dependent. The chosen probability will be implemented by drawing one card from a set of 10 cards numbered from 1 to 10. If p is 0, Allocation 1 will not be implemented and Allocation 2 will be implemented regardless of the card drawn. If p is 0.1, Allocation 1 will be implemented if the card drawn is numbered 1; otherwise, Allocation 2 will be implemented. If p is 0.2, Allocation 1 will be implemented if the card drawn is numbered either 1 or 2; otherwise, Allocation 2 will be implemented, and so on.

We include 11 pairs of allocations that vary in inequality and efficiency in the allocation triangle as illustrated in Fig. 1. In menus $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$, the total pie (efficiency) is fixed and inequality is reversed between the two allocations. Menu $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ follows Machina (1989) and is tested in Sandroni et al. (2013).

Intuitively, subjects choose (20, 0) over (0, 20) in a deterministic situation due to selfishness, whereas preference for ex-ante fairness drives the subjects to randomize between the two allocations. With menu $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$, we can test whether subjects are more or less willing to randomize when the inequality is less extreme.

In menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$, one allocation has disadvantageous inequality for the dictator while the other allocation shrinks the pie to zero for both players. In menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, one allocation has disadvantageous inequality for the dictator while the other allocation shrinks the pie by half for the dictator. Intuitively, inequality aversion predicts preference for (0, 0) over (0, 20), whereas efficiency concern predicts the opposite. Choosing to randomize between the two allocations would reflect a mixture of both equality and efficiency concerns.[6] With menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, we can test the effect of varying efficiency on subjects' willingness to randomize. Similarly, we include menus $\begin{pmatrix} 4, & 16 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 4, & 16 \\ 2, & 8 \end{pmatrix}$, which further incorporates the selfishness concern.

We introduce four more menus $\begin{pmatrix} 20, & 0 \\ 0, & 0 \end{pmatrix}$, $\begin{pmatrix} 16, & 4 \\ 0, & 0 \end{pmatrix}$, $\begin{pmatrix} 20, & 0 \\ 10, & 0 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 8, & 2 \end{pmatrix}$, which shares similar intuitions to the menus discussed above. However, we switch the payoffs between the dictator and the recipient to impose advantageous inequality for the dictator. This switch in payoffs enables us to compare between disadvantageous and advantageous inequality. The last menu is $\begin{pmatrix} 10, & 10 \\ 0, & 0 \end{pmatrix}$, in which the subjects have equal payoffs of 10, and the alternative is 0 for both of them. This menu is included as a control because any mixture of selfishness, equality, or efficiency concern would not predict randomization.

We recruited 157 students through an advertisement posted on a web-based platform, the Integrated Virtual Learning Environment at the National University of Singapore (NUS). The experiment consisted of six sessions with 20–30 subjects in each session. Upon arriving at the experimental venue, subjects were given the consent form approved by the NUS institutional review board. Subsequently, general instructions were read aloud to the subjects. We demonstrated a few examples and then gave several exercises to the subjects to practice. After ensuring that the subjects fully understood the tasks, subjects began with their decisions-making tasks. The 11 menus were randomly presented to each subject. Most of the subjects completed the tasks within ten minutes. Each subject received a participation fee of SGD10 (approximately USD7.40) at the end of the session. We randomly selected one participant in each session to implement one of her 11 choices and matched her choice with one randomly selected subject in the session.

[6] This view echoes the intuition in the convex ultimatum game in Andreoni et al. (2003), in which the responders can choose to shrink the pie instead of accepting or rejecting the offer.

2.2 Experiment II

In Experiment I, we incentivize subjects by randomly selecting one participant in each session to implement one of her 11 choices. Although this so-called random lottery mechanism has been widely used and has a number of advantages in data collections, its validity could be of concern (see, Wakker 2007 for a related discussion), especially given our interest in ex-ante preference. To address this concern and to check the robustness of the results in Experiment I, we conduct Experiment II in which the subjects play the role of either the dictator or the recipient. In addition, each dictator makes one single choice in the experiment, which is implemented with certainty with real incentive. Each recipient is matched with a dictator and makes no choice in the experiment. We include two menus in Experiment II: $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$. These two menus allow the decision maker to randomize to trade-off between equality and efficiency motives, as explained earlier. Other aspects of Experiment II are the same as Experiment I.

We recruited 188 subjects for this experiment. Ninety-four subjects played the role of dictators: 45 subjects were given the menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and 49 subjects were given the menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$. The other 94 subjects were recipients and were matched to the 94 dictators. We had seven sessions and mixed the two menus in each session to control for the possible session effect. Each subject received a show up fee of SGD5 (approximately USD3.70), in addition to the payment based on the choice of the dictators.

3 Theoretical framework and predictions

In this section, we first present the theoretical framework including ex-post preference, ex-ante preference and combinational preference. We then analyze the theoretical predictions for different menus.

3.1 Theoretical framework

Denote $\Phi\left(\left(x_1, x_2\right), p; \left(y_1, y_2\right), 1-p\right)$ a general utility function for a contingent allocation $\left(\left(x_1, x_2\right), p; \left(y_1, y_2\right), 1-p\right)$, which yields the deterministic allocation (x_1, x_2) with probability p and (y_1, y_2) with probability $1-p$.

When there is no risk involved, i.e., p equals either 0 or 1, Φ reduces to deterministic social preference utility, denoted by $U\left(x_1, x_2\right)$ and $U\left(y_1, y_2\right)$, in which U captures preference for fairness that can admit various forms including Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Charness and Rabin (2002), Andreoni and Miller (2002), and Cox et al. (2007). When the decision maker is purely selfish, Φ reduces to individual utility under risk, denoted by $\Theta\left(x_1, p; y_1, 1-p\right)$ in which Θ is a risk aggregator function that can adopt either expected utility or non-expected utility models (i.e., Kahneman and Tversky 1979; Quiggin 1982; Chew 1983; Dekel 1986).

When risk preference and social preference are intertwined, two types of preference specifications are of particular interest. One approach is to first evaluate each ex-post contingent allocation and then aggregates the overall utility with risk preference. The resulting utility is given by:

$$\Phi_{\text{ex-post}} = \Theta\left(U\left(x_1, x_2\right), p; U\left(y_1, y_2\right), 1-p\right).$$

In the representation, $U\left(x_1, x_2\right)$ and $U\left(y_1, y_2\right)$ are the utilities of each deterministic allocation with risk preference utility Θ aggregating the overall utility of the contingent allocation. $\Phi_{\text{ex-post}}$ captures preference for ex-post fairness since the social preference utility is applied for evaluating ex-post allocations. Note that this ex-post preference $\Phi_{\text{ex-post}}$ is weakly separable in the risk dimension, i.e., the marginal rate of substitution between x_1 and x_2 is independent of the values of y_1 and y_2, and vice versa. In addition, the social preference utility U is assumed to be state independent, i.e., the same U applies for the two different states.

Another approach is to first evaluate the contingent risk for each player and then assess the overall utility using social preference. The resulting utility is given by:

$$\Phi_{\text{ex-ante}} = U\left(\Theta\left(x_1, p; y_1, 1-p\right), \Theta\left(x_2, p; y_2, 1-p\right)\right).$$

In the representation, $\Theta\left(x_1, p; y_1, 1-p\right)$ and $\Theta\left(x_2, p; y_2, 1-p\right)$ are the respective utilities for the risk each individual faces, and social preference utility U aggregates the overall utility for the contingent allocation. $\Phi_{\text{ex-ante}}$ captures preference for ex-ante fairness since the social preference utility is applied for evaluating ex-ante allocation. Similarly, this ex-ante preference $\Phi_{\text{ex-ante}}$ is weakly separable in the dimension of individual, i.e., the marginal rate of substitution between x_1 and y_1 is independent of the values of x_2 and y_2, and vice versa. In addition, the same risk preference (Θ function) is applied to evaluate the payoffs of both dictator and recipient.

Lastly, a combinational preference Φ_c is given by:

$$\begin{aligned}&\Phi_c(\Theta\left(U\left(x_1, x_2\right), p; U\left(y_1, y_2\right), 1-p\right),\\&\quad U\left(\Theta\left(x_1, p; y_1, 1-p\right), \Theta\left(x_2, p; y_2, 1-p\right)\right),\end{aligned}$$

which incorporates both ex-ante preference $\Phi_{\text{ex-ante}}$ and ex-post preference $\Phi_{\text{ex-post}}$ with an overall aggregate function Φ_c. Note that Φ_c is not separable in either dimension of risk or individual. In the sequel, we will consider certain restrictions on U, Θ and Φ_c to deliver several predictions of the above preference specifications, and then briefly discuss the predictions of general functional forms.

3.2 Theoretical predictions

Consider ex-post preference $\Phi_{\text{ex-post}} = \Theta\left(U\left(x_1, x_2\right), p; U\left(y_1, y_2\right), 1-p\right)$. If risk preference Θ respects first-order stochastic dominance, it implies that $\Theta\left(U\left(x_1, x_2\right)\right) >$

$\Theta\left(U\left(x_1, x_2\right), p; U\left(y_1, y_2\right), 1-p\right) > \Theta\left(U\left(y_1, y_2\right)\right)$ for $p \in (0, 1)$ if $U\left(x_1, x_2\right) > U\left(y_1, y_2\right)$. Thus, ex-post preference $\Phi_{\text{ex-post}}$ predicts corner choices in all menus and we have the following prediction.[7]

Prediction 1 *Ex-post preference predicts corner choices if Θ respects first-order stochastic dominance.*

Notably, first-order stochastic dominance is respected by most commonly used models, including expected utility, rank-dependent utility (Quiggin 1982) and those adopting the betweenness approach (Chew 1983; Dekel 1986). This implication is in line with the independence property discussed in Fudenberg and Levine (2012).

For ex-ante preference $\Phi_{\text{ex-ante}} = U\left(\Theta\left(x_1, p; y_1, 1-p\right), \Theta\left(x_2, p; y_2, 1-p\right)\right)$, we first consider the case with Θ being linear in both probabilities and outcomes, i.e., ex-ante preference defined on expected allocations for the two players as $U\left(px_1 + (1-p)\, y_1, px_2 + (1-p)\, y_2\right)$. We consider the properties of U including *proportional monotonicity* and strict concavity. We say that U is proportionally monotonic if $U\left(rx_1, rx_2\right)$ is monotonically increasing or decreasing in r (see Fig. 2). We have the following prediction for proportionally monotonic U.

Prediction 2A *Ex-ante preference predicts corner choices in menus* $\begin{pmatrix} x_1, & x_2 \\ rx_1, & rx_2 \end{pmatrix}$ *with* $r \epsilon [0, 1)$ *if U is proportionally monotonic.*

The utility $\Phi_{\text{ex-ante}}\left(\left(x_1, x_2\right), p; \left(rx_1, rx_2\right), 1-p\right)$ is given by $U\left(\left(r + p\left(1-r\right)\right) x_1, \left(r + p\left(1-r\right)\right) x_2\right)$. Therefore, a dictator admitting ex-ante preference with a proportionally monotonic utility U chooses corner probabilities in menus $\begin{pmatrix} x_1, & x_2 \\ rx_1, & rx_2 \end{pmatrix}$ including $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$. Proportional monotonicity is widely admitted in various models including Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Charness and Rabin (2002), Andreoni and Miller (2002), and Cox et al. (2007).[8]

The other common property of U is strict concavity (see Fig. 2). We have the following prediction for strictly concave U.

Prediction 2B *Ex-ante preference predicts that a decision maker who chooses $p = 0$ in menu* $\begin{pmatrix} x_1, & x_2 \\ rx_1, & rx_2 \end{pmatrix}$ *will also choose $p' = 0$ in menu* $\begin{pmatrix} x_1, & x_2 \\ r'x_1, & r'x_2 \end{pmatrix}$ *with $r' > r$ if U is strictly concave.*

[7] It is possible to have $U\left(x_1, x_2\right) = U\left(y_1, y_2\right)$ in some menus in our setup, and ex-post preference remains silent in these cases.

[8] Fehr and Schmidt (1999) admit the form $x_1 - \alpha \max\left\{x_1 - x_2, 0\right\} - \beta \max\left\{x_2 - x_1, 0\right\}$. Bolton and Ockenfels (2000) take the form $x_1 - \alpha \max\left\{\left(x_1 - x_2\right) / \left(x_1 + x_2\right), 0\right\} - \beta \max\left\{\left(x_2 - x_1\right) / \left(x_1 + x_2\right), 0\right\}$. Charness and Rabin (2002) admit the form $\left(1-\gamma\right) x_1 + \gamma\left(\delta \min\left\{x_1, x_2\right\} + \left(1-\delta\right)\left(x_1 + x_2\right)\right)$. Andreoni and Miller (2002) take the form $\left(\delta x_1^{\alpha} + (1-\delta)\, x_2^{\alpha}\right)^{1/\alpha}$. Cox et al. (2007) admit the form $\left(x_1^{\alpha} + \theta x_2^{\alpha}\right)/\alpha$. In all of these models, varying p in $\left(px_1, px_2\right)$ does not change the relative rank between px_1 and px_2, and thus would not change the parameters in these models when evaluating $\left(x_1, x_2\right)$ and $\left(px_1, px_2\right)$. Given the linearity in the first three functional forms and the CES form in the latter two, all of these models exhibit proportional monotonicity.

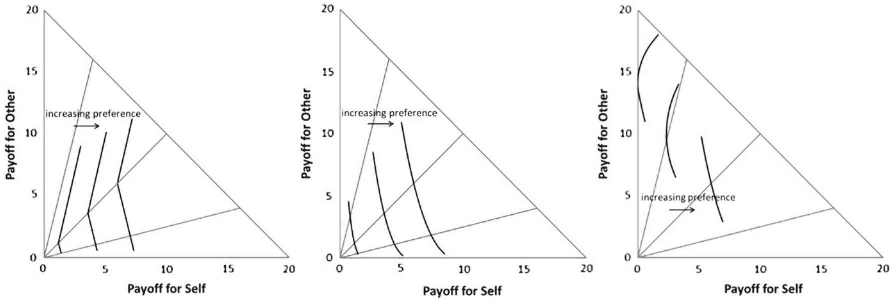

Fig. 2 Illustration of properties of social preferences. From *left* to *right*, we show the indifference *curves* that represent (i) proportionally monotonic and not strictly convex preference; (ii) proportionally monotonic and strictly convex preference; and (iii) not proportionally monotonic and strictly convex preference

This prediction is due to the fact that a strictly concave U admits unique optimal allocation along any interval in the allocation triangle.[9] Given two menus $\begin{pmatrix} x_1, & x_2 \\ rx_1, & rx_2 \end{pmatrix}$ and $\begin{pmatrix} x_1, & x_2 \\ r'x_1, & r'x_2 \end{pmatrix}$ with $r' > r$, suppose a decision maker chooses $p = 0$ in the first menu and $p' > 0$ in the second, which results in ex-ante allocations (rx_1, rx_2) and $\left((r' + p'\left(1 - r'\right))x_1, (r' + p'\left(1 - r'\right))x_2\right)$, respectively. Then, a convex combination of these two ex-ante allocations can generate $\left(r'x_1, r'x_2\right)$, which in turn should be preferred to $\left((r' + p'\left(1 - r'\right))x_1, (r' + p'\left(1 - r'\right))x_2\right)$ since U is strictly concave and (rx_1, rx_2) is preferred to $\left((r' + p'\left(1 - r'\right))x_1, (r' + p'\left(1 - r'\right))x_2\right)$. This contradicts the optimality of choosing $p' > 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ r'x_1, & r'x_2 \end{pmatrix}$. Therefore, the decision maker must also choose $p' = 0$ in $\begin{pmatrix} x_1, & x_2 \\ r'x_1, & r'x_2 \end{pmatrix}$. In our experimental setting, this prediction suggests that subjects choosing $p = 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ will also choose $p = 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$. Strict concavity appears in several social preference specifications admitting the CES form, such as Andreoni and Miller (2002), Karni and Safra (2002), and Cox et al. (2007).

The predictions of ex-ante preference are not straightforward when Θ admits a more general form, as the ex-ante allocation derived from $(\Theta\left(x_1, p; y_1, 1 - p\right),$ $\Theta\left(x_2, p; y_2, 1 - p\right))$ usually does not lie on the line extended by (x_1, x_2) and (y_1, y_2).[10] Nevertheless, for menus like $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, risk attitude does

[9] Levati et al. (2014) test the single-peakedness property of social preference, which is implied by concavity.

[10] Given that Θ admits the expected utility with a homogeneous utility function u, the ex-ante allocation for menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ under any p, which is $\left(u^{-1}\left(pu\left(x_1\right)\right), u^{-1}\left(pu\left(x_2\right)\right)\right)$, lies on the line extended by (x_1, x_2) and $(0, 0)$. Krawczyk and Le Lec (2014) suggest that several observations on ex-ante preferences in Brock et al. (2013) can be rationalized by risk aversion.

not play a role for the dictator since the payoffs for the dictator are always 0. Considering risk attitude, the ex-ante allocations in these menus still lie on the vertical line of the triangle and the preceding predictions remain valid.[11] In particular, given a general Θ, proportional monotonicity in U still predicts corner choices in menus $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$, $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, $\begin{pmatrix} 20, & 0 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 20, & 0 \\ 10, & 0 \end{pmatrix}$, while strict concavity predicts that we should observe (weakly) more choices of $p = 0$ when switching from menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ to menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, and from menu $\begin{pmatrix} 20, & 0 \\ 0, & 0 \end{pmatrix}$ to menu $\begin{pmatrix} 20, & 0 \\ 10, & 0 \end{pmatrix}$.

When the combinational preference $\Phi_c\left(\Phi_{\text{ex-ante}}, \Phi_{\text{ex-post}}\right)$ is concerned, we have the following prediction.

Prediction 3 *The combinational preference predicts corner choices in menus* $\begin{pmatrix} x_1, & x_2 \\ rx_1, & rx_2 \end{pmatrix}$ *with* $r \in [0, 1)$ *if the following assumptions jointly hold: (i)* Θ *is linear in probabilities and outcomes; (ii)* U *admits proportional monotonicity; and (iii)* Φ_c *is monotonic in both arguments.*

The intuition for this prediction is as follows. For a contingent allocation $((x_1, x_2), p; (rx_1, rx_2), 1-p)$, linearity of Θ implies that ex-post preference $\Phi_{\text{ex-post}} = \Theta(U(x_1, x_2), p; U(rx_1, rx_2), 1-p)$ is monotonic in p. Moreover, linear Θ together with proportionally monotonic U imply that ex-ante preference $\Phi_{\text{ex-ante}} = U(\Theta(x_1, p; rx_1, 1-p), \Theta(x_2, p; rx_2, 1-p))$ is also monotonic in p. Therefore, the overall utility $\Phi_c\left(\Phi_{\text{ex-ante}}, \Phi_{\text{ex-post}}\right)$ will be monotonic in p given the monotonicity of Φ_c, which in turn predicts corners choices in all menus $\begin{pmatrix} x_1, & x_2 \\ rx_1, & rx_2 \end{pmatrix}$.

One example of such a combinational model satisfying all these conditions, with U admitting the Fehr and Schmidt (1999) specification, Θ taking the expected value form, and Φ_c a weighted average of ex-post and ex-ante utilities, is discussed in Fudenberg and Levine (2012) and axiomatized in Saito (2013).[12]

4 Results and analyses

4.1 Basic patterns

In this subsection, we summarize the basic choice patterns in Experiment I with respect to the theoretical predictions and check the robustness of the patterns in Experiment II. Figure 3 summarizes the percentage of subjects choosing $p = 0$, $p = 1$ and interior

[11] Similar results hold for the two menus on the bottom line of the triangle.

[12] Rohde (2010) axiomatizes deterministic Fehr–Schmidt specification. See Saito (2013) for discussions on the connection between Rohde (2010) and Saito (2013). Neilson (2006) considers the applicability of the standard separability axiom for both risk and other-regarding preferences and the resulting representation coincides with Fehr–Schmidt specification for other-regarding preferences, and prospect theory for risk preferences. We show in "Appendix 2" that the incompatibility between interior choice and Saito (2013) is due to the axioms of quasi-comonotonic independence and dominance.

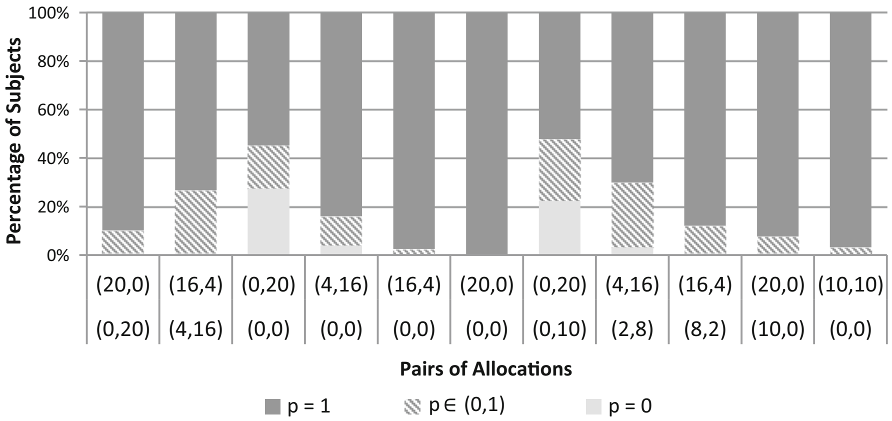

Fig. 3 Percentage of subjects choosing different types of probability for each menu

probabilities for each of the 11 menus in Experiment I (see also Table 2 of "Appendix 1"). The first observation is as follows.

Observation 1 *Substantial proportions of subjects choose interior probabilities across menus.*

The proportion of interior choices varies across different menus from 0 to 26.8%. For instance, no subject chooses interior p in menu $\begin{pmatrix} 20, & 0 \\ 0, & 0 \end{pmatrix}$, whereas 26.8% of the subjects choose interior p in menu $\begin{pmatrix} 4, & 16 \\ 2, & 8 \end{pmatrix}$. In the control menu $\begin{pmatrix} 10, & 10 \\ 0, & 0 \end{pmatrix}$, 96.8% of the subjects choose $p = 1$, no one chooses $p = 0$, and 3.2% of the subjects choose interior p. If the proportion of interior choices in other menus is significantly higher than that in the control menu, it is unlikely to be driven by simple choice error.

For the two menus along the hypotenuse $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$, the majority of the subjects choose $p = 1$, one subject chooses $p = 0$ in menu $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$, and another subject chooses $p = 0$ in menu $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$.[13] The levels of interior probability choices are 9.6 and 26.1%, respectively, which are significantly more than 3.2% in the control menu $\begin{pmatrix} 10, & 10 \\ 0, & 0 \end{pmatrix}$ (proportion test $p < 0.001$, two-tailed). This observation is similar to that in Sandroni et al. (2013) and supports the intuition in Machina's thought experiment in the context of social preference. The average probabilities for these interior choices are 0.673 and 0.741, both significantly larger than

[13] These two incidences of choosing $p = 0$ are similar to those giving more than 50% in the standard dictator game (e.g., Camerer 2003). Our results are robust to the exclusion of these two subjects. If we consider the five subjects choosing interior p for $\begin{pmatrix} 10, & 10 \\ 0, & 0 \end{pmatrix}$ as noises, our results are also robust to the exclusion of these five subjects.

0.5 ($p < 0.001$). This result suggests that although the dictator prefers to share the probability with the recipient, she would prefer to give herself a more favorable probability. Moreover, the proportion of interior probability choice in menu $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$ is significantly higher than that in menu $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ ($p < 0.001$), which suggests that subjects prefer randomization when inequity is less extreme.

When facing disadvantageous inequality, the percentage of interior choices is 17.8% in menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and 25.5% in menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, with corresponding mean interior probabilities of 0.414 and 0.493, respectively. Similarly, the percentage of interior choices is 12.1% in menu $\begin{pmatrix} 4, & 16 \\ 0, & 0 \end{pmatrix}$ and 26.8% in menu $\begin{pmatrix} 4, & 16 \\ 2, & 8 \end{pmatrix}$, with corresponding mean interior probabilities of 0.515 and 0.595, respectively.

By contrast, when subjects face advantageous inequality, the percentage of interior choices is lower, 0% in menu $\begin{pmatrix} 20, & 0 \\ 0, & 0 \end{pmatrix}$ and 7% in menu $\begin{pmatrix} 20, & 0 \\ 10, & 0 \end{pmatrix}$, 2.5% in menu $\begin{pmatrix} 16, & 4 \\ 0, & 0 \end{pmatrix}$ and 11.5% in menu $\begin{pmatrix} 16, & 4 \\ 8, & 2 \end{pmatrix}$. The proportion test shows the presence of significant differences for all the comparisons of disadvantageous inequality and advantageous inequality (Table 2 of "Appendix 1"). The mean interior probability is 0.501 for the four menus with disadvantageous inequality, and 0.716 for the four menus with advantageous inequality. These results suggest that although some subjects choose randomization under advantageous inequality, the chosen probabilities are higher than those in the menus with disadvantageous inequality.

We test the robustness of the observation in Experiment II. In Experiment II, the percentage of interior choices is 37.8% in menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and 26.5% in menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$. This result suggests that the observed preference for randomization is robust across different elicitation mechanisms. We adopt a two-sample proportion test and find that the proportions are not significantly different between the two experiments for menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ ($p > 0.183$), as well as for menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$ ($p > 0.208$). [14]

In terms of theoretical predictions, the prevalence of interior choices is incompatible with ex-post preference respecting first-order stochastic dominance as in Prediction 1. Moreover, a substantial proportion of subjects choose interior probabilities in menus $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$, $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, $\begin{pmatrix} 4, & 16 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 4, & 16 \\ 2, & 8 \end{pmatrix}$. These results cannot be accounted for by ex-ante preference permitting proportional monotonicity as in Prediction 2A, or a combinational preference concurrently permitting stochastic dominance, proportional monotonicity, and monotonicity as in Prediction 3.

[14] Alternatively, we use the Chi-square test, which yields similar statistics.

To examine Prediction 2B, we test whether the subjects who choose $p = 0$ in $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ continue to choose $p = 0$ in $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$. The second observation is as follows.

Observation 2 *A substantial proportion of subjects choose* $p = 0$ *in menu* $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ *and choose* $p > 0$ *in menu* $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$.

In Experiment I, among the subjects who choose $p = 0$ in menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$, 41.7% of them choose $p > 0$ in menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$. Similarly, among the subjects who choose $p = 0$ in menu $\begin{pmatrix} 4, & 16 \\ 0, & 0 \end{pmatrix}$, 66.7% of them choose $p > 0$ in menu $\begin{pmatrix} 4, & 16 \\ 2, & 8 \end{pmatrix}$. This result is incompatible with ex-ante preference with a strictly concave U, which predicts subjects who choose $p = 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ will also choose $p = 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$. For Experiment II with between-subject design, we compare the proportions of subjects choosing $p = 0$ in menus $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$ due to the between-subject design. The proportion is 22.2% in menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$, which is more than the proportion of 18.4% in menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$. This result complements the observation in Experiment I and similarly suggests the violation of ex-ante preference with a strictly concave U.[15]

[15] Similar to the intuition in Prediction 2B, it can be shown that a more restrictive prediction of strict concavity is that the proportion of subjects choosing $p \leq 0.5$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ equals the proportion of subjects choosing $p = 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$. In Experiment I, the proportion of subjects choosing $p = 0$ in $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$ is 22.3%, which is significantly less than 42.7%, the corresponding proportion of subjects choosing $p \leq 0.5$ in $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$ (proportion test, $p < 0.001$). Similarly, the proportion of subjects choosing $p = 0$ in $\begin{pmatrix} 4, & 16 \\ 2, & 8 \end{pmatrix}$ is 2.3%, which is significantly less than 10.2%, the corresponding proportion of subjects choosing $p \leq 0.5$ in $\begin{pmatrix} 4, & 16 \\ 0, & 0 \end{pmatrix}$ (proportion test, $p < 0.013$). For Experiment II, the proportion of subjects choosing $p = 0$ is 18.4% in menu $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$, which is significantly less than 48.9%, the proportion of subjects choosing $p \leq 0.5$ in menu $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ (proportion test, $p < 0.002$).

4.2 Individual analysis

We proceed to conduct an individual-level analysis by characterizing some basic behavioral patterns of the subjects. As Experiment I is conducted using a within-subject design and Experiment II is conducted using a between-subject design, the individual analysis is only feasible for Experiment I.

Seventy-five subjects (47.8%) choose corner probabilities for all of the menus, and the behavioral patterns of these subjects are consistent with ex-post preference.[16] Sixty-nine subjects (44.0%) make consistent choices in menus $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$, as well as in menus $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$. That is, if the chosen probability in menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ generates an ex-ante allocation that lies in between (x_1, x_2) and $(0.5x_1, 0.5x_2)$, then the chosen probability in menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$ generates the same ex-ante allocation.[17] These patterns can be explained by ex-ante preference. There are intersections of these two types. Sixty-three subjects (40.1%) always choose corner probabilities, and they choose the same corner probabilities in menus $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$.

Besides the 63 subjects above, we have an additional 16 subjects (10.2%) choosing corner probabilities in all of the menus except for menus $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$. The combinational preference with Fehr–Schmidt specification in Saito (2013) can rationalize these patterns as it does not impose corner choice on the hypotenuse of the allocation triangle.

In sum, the behavior patterns of 91 subjects (58.0%) could be accounted for by the various models mentioned in Sect. 3, whereas the remaining 66 subjects (42.0%) do not belong to any type. These patterns suggest that the incompatibility between theoretical assumptions and observed patterns persists at the individual level.[18]

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[16] We also check whether the chosen probabilities result in violations of transitivity. For instance, $p = 1$ in $\begin{pmatrix} 20, & 0 \\ 0, & 0 \end{pmatrix}$, $p = 0$ in $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$, and $p = 0$ in $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ violates transitivity. No subject violates transitivity.

[17] If the generated ex-ante allocation does not lie in between (x_1, x_2) and $(0.5x_1, 0.5x_2)$, then the chosen probability must be 0 in menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$ to be consistent with strict convexity. Moreover, with 11 discrete probabilities in each menu, sometimes the chosen probabilities cannot generate exactly the same ex-ante allocation. We allow for a difference of 0.05 in the probabilities when counting for ex-ante type.

[18] We check the behavioral patterns consistent with pure selfishness, efficiency, and inequality aversion motives. The selfish type chooses 1 in all the menus except for menus $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$; 86 subjects (54.8%) belong to this group. The efficient type chooses 1 in all of the menus to maximize efficiency except for menus $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$; 67 subjects (42.7%) belong to this group. In

4.3 Further discussions

Overall, the observed choice patterns are incompatible with either ex-ante preference or ex-post preference under some regular assumptions. In addition, interior choices are also incompatible with certain assumptions in the combinational preference $\Phi_c\left(\Phi_{\text{ex-ante}}, \Phi_{\text{ex-post}}\right)$. Generally speaking, it is relatively easier to give up proportional monotonicity in U while maintaining dominance in $\Phi_{\text{ex-post}}$ and monotonicity in Φ_c. For instance, Andreoni et al. (2003) observe evidence against proportional monotonicity in the responder preferences in a convex deterministic ultimatum game and suggest that the intention model in Rabin (1993) can display such non-monotonicity. As our setup of the dictator game does not involve reciprocity, Rabin's model does not seem to account for the observed violation of proportional monotonicity.

One can have non-proportional monotonicity using a nonlinear trade-off between selfishness and inequality aversion as mentioned in Fehr and Schmidt (1999), e.g., $x_1 - \alpha\left(\max\left\{x_1 - x_2, 0\right\}\right)^2 - \beta\left(\max\left\{x_2 - x_1, 0\right\}\right)^2$, where α and β are the respective inequality aversion parameters when a dictator faces advantageous and disadvantageous inequality. Ex-ante preference with a social preference utility U admitting the above form can accommodate the interior choices as in Observation 1 because of the convexity in inequality aversion. With $\alpha < \beta$, the inequality aversion incentive becomes stronger when switching from the lower to the upper part of the triangle. This in turn can induce more interior choices in the upper part of the triangle. Nevertheless, such a preference is incompatible with the interior choices in menus $\begin{pmatrix} 0, & 20 \\ 0, & 0 \end{pmatrix}$ and $\begin{pmatrix} 0, & 20 \\ 0, & 10 \end{pmatrix}$ because selfish concern is not involved in either menu. Choosing $p > 0$ in these two menus indicates the potential existence of an efficiency concern, which can be captured by an additional term $\delta\left(x_1 + x_2\right)$, in which δ measures the relative weight for efficiency concern. This results in an expression of $x_1 - \alpha\left(\max\left\{x_1 - x_2, 0\right\}\right)^2 - \beta\left(\max\left\{x_2 - x_1, 0\right\}\right)^2 + \delta\left(x_1 + x_2\right)$.[19] This utility function is globally concave, and hence ex-ante preference with this social preference utility is still incompatible with Observation 2. Incorporating ex-post preference may resolve this incompatibility. Intuitively, ex-post preference respecting dominance

contrast, no subject belongs to the inequality aversion type, who is supposed to minimize the inequality. The prediction of the selfish type is independent of ex-ante and ex-post concerns, i.e., maximizing either the ex-ante or ex-post selfishness utility results in the same predictions. Similarly, ex-ante and ex-post efficiency concerns predict the same choice patterns. The implication for inequality aversion is slightly different, and ex-post and ex-ante inequality-averse subjects may behave differently in menus $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$. For instance, a quadratic ex-ante inequality-averse agent chooses 0.5, whereas a quadratic ex-post inequality-averse subject is indifferent among all of the probabilities.

[19] Efficiency and inequality aversion cannot account for the overall behavior because efficiency constantly predicts the choice of $p = 1$ in all menus except for $\begin{pmatrix} 20, & 0 \\ 0, & 20 \end{pmatrix}$ and $\begin{pmatrix} 16, & 4 \\ 4, & 16 \end{pmatrix}$, whereas inequality is minimized along the 45° line in the allocation triangle. The presence of only efficiency and inequality aversion motives would imply that we should observe more corner choices for menus lying on the 45° line, which is incompatible with the overall data.

predicts corner choices, which drives subjects away from choosing interior probabilities. When switching from menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ to menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$, the incentive of choosing corner probabilities due to preference for ex-post fairness may diminish. Hence, the subjects could exhibit less willingness to choose $p = 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$, as in Observation 2. We provide detailed analyses of this proposed behavioral model in "Appendix 3".

5 Conclusion

We build on recent theoretical and experimental works on ex-ante and ex-post social preferences, and provide an experimental setup that enables us to systematically explore probabilistic social preferences inside the allocation triangle. We reveal evidence against a number of weak assumptions, including first-order stochastic dominance, proportional monotonicity, and convexity. We propose a combinational preference with a modified Fehr and Schmidt (1999) specification to capture the observed patterns. More generally, our study sheds light on future investigations on social preferences under risk, especially on those combining both ex-ante and ex-post concerns.

Appendix 1: Appended tables

See Tables 2 and 3.

Table 2 Summary statistics

Menu	Allocation 1		Allocation 2		% $p = 1$	% $p = 0$	% $p \in (0,1)$	Mean p_1	Mean p_2
	x_1	x_2	y_1	y_2					
1	20	0	0	20	0.90	0.01	0.10	0.96	0.67
2	16	4	4	16	0.73	0.01	0.26	0.93	0.74
3	0	20	0	0	0.55	0.27	0.18	0.62	0.41
4	4	16	0	0	0.84	0.04	0.12	0.90	0.52
5	16	4	0	0	0.97	0.00	0.03	0.99	0.55
6	20	0	0	0	1.00	0.00	0.00	1.00	0.00
7	0	20	0	10	0.52	0.22	0.25	0.65	0.49
8	4	16	2	8	0.70	0.03	0.27	0.85	0.56
9	16	4	8	2	0.88	0.01	0.11	0.97	0.76
10	20	0	10	0	0.92	0.01	0.07	0.98	0.74
11	10	10	0	0	0.97	0.00	0.03	0.99	0.66

Columns 6, 7 and 8 summarize percentage of choosing probability 1 for Allocation 1, the percentage of subjects choosing probability 0 for Allocation 1, percentage of choosing interior probability for Allocation 1. Column 9 summarizes the mean probability to implement Allocation 1, and Column 10 summarizes the mean probability for interior probabilities to implement Allocation 1

Table 3 The comparison of interior choices

Menu	x_1	x_2	y_1	y_2	1	2	3	4	5	6	7	8	9	10
1	20	0	0	20										
2	16	4	4	16	0.00									
3	0	20	0	0	0.03	0.08								
4	4	16	0	0	0.47	0.00	0.16							
5	16	4	0	0	0.01	0.00	0.00	0.00						
6	20	0	0	0	0.00	0.00	0.00	0.00	0.04					
7	0	20	0	10	0.00	0.90	0.10	0.00	0.00	0.00				
8	4	16	2	8	0.00	0.90	0.06	0.00	0.00	0.00	0.80			
9	16	4	8	2	0.58	0.00	0.11	0.86	0.00	0.00	0.00	0.00		
10	20	0	10	0	0.41	0.00	0.00	0.13	0.06	0.00	0.00	0.00	0.17	
11	10	10	0	0	0.02	0.00	0.00	0.00	0.74	0.02	0.00	0.00	0.01	0.12

This table presents the p values of proportion tests for comparing the percentage of interior choice across menus

Appendix 2: Analysis of axioms in Saito (2013)

We demonstrate that the two axioms, namely Quasi-comonotonic Independence and Dominance, imply the monotonicity of the combination preference in p for $((x_1, x_2), p; (0, 0), (1 - p))$. The two axioms are as follows:

Quasi-comonotonic Independence: For all $p \in (0, 1]$, and (x_1, x_2), (y_1, y_2), (z_1, z_2) that are pairwise comonotonic, $(x_1, x_2) \succeq (y_1, y_2)$ if and only if $p(x_1, x_2) + (1 - p)(z_1, z_2) \succeq p(y_1, y_2) + (1 - p)(z_1, z_2)$.

Dominance: Given two contingent allocations $((x_1, x_2), p; (z_1, z_2), (1 - p))$ and $((y_1, y_2), q; (w_1, w_2), (1 - q))$, $\mathrm{E}_{(px_1+(1-p)z_1, px_2+(1-p)z_2)} \geq \mathrm{E}_{(qy_1+(1-q)w_1, qy_2+(1-q)w_2)}$ and $p\mathrm{E}_{(x_1,x_2)} + (1 - p)\mathrm{E}_{(z_1,z_2)} \geq q\mathrm{E}_{(y_1,y_2)} + (1 - q)\mathrm{E}_{(w_1,w_2)}$ imply $((x_1, x_2), p; (z_1, z_2), (1 - p)) \succeq ((x_1, x_2), q; (w_1, w_2), (1 - q))$, where E denotes the equality equivalent for an allocation, i.e., $\left(\mathrm{E}_{(x_1,x_2)}, \mathrm{E}_{(x_1,x_2)}\right) \sim (x_1, x_2)$.

Intuitively, the Quasi-comonotonic Independence axiom states that the preference over deterministic allocations satisfies the usual independence axiom separately in the upper and lower parts of the triangle, which directly implies that the preference over deterministic allocations is monotonic along straight lines passing through 0 because all of the allocations along the line belong to either the upper or lower part of the triangle. The Quasi-comonotonic Independence axiom taken together with the Dominance axiom implies the monotonicity of combinational preference. Formally, suppose $(x_1, x_2) \succ (0, 0)$. Then the Quasi-comonotonic Independence implies $p(x_1, x_2) + (1 - p)(0, 0) \succeq q(x_1, x_2) + (1 - q)(0, 0)$ for $p \geq q$. Therefore, we have $\mathrm{E}_{(px_1, px_2)} \geq \mathrm{E}_{(qx_1, qx_2)}$, which in turn implies $((x_1, x_2), p; (0, 0), (1 - p)) \succeq ((x_1, x_2), q; (0, 0), (1 - q))$ with Dominance. In sum, the two axioms imply corner choices in all the menus $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$. The argument in situation $(x_1, x_2) \prec (0, 0)$ is similar.

Appendix 3: Analysis of the proposed behavioral model

Assume the utility function for deterministic allocations takes the following form:

$$x_1 + \delta\left(x_1 + x_2\right) - \alpha\left(\max\{x_1 - x_2, 0\}\right)^2 - \beta\left(\max\{x_2 - x_1, 0\}\right)^2,$$

and the combinational utility is a weighted average $\lambda\Phi_{\text{ex-ante}} + (1-\lambda)\,\Phi_{\text{ex-post}}$. For menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ with $x_1 < x_2$, the utility for an interior chioce p is

$$\lambda\left(px_1 + \delta p\left(x_1 + x_2\right) - \beta\left(p\left(x_1 - x_2\right)\right)^2\right) \\ + (1-\lambda)\left(p\left(x_1 + \delta\left(x_1 + x_2\right) - \beta\left(x_1 - x_2\right)^2\right) + (1-p) * 0\right)$$

and we obtain the following FOC characterizing the optimal choice of p:

$$\lambda\left(x_1 + \delta\left(x_1 + x_2\right) - 2\beta p\left(x_1 - x_2\right)^2\right) \\ + (1-\lambda)\left(x_1 + \delta\left(x_1 + x_2\right) - \beta\left(x_1 - x_2\right)^2\right) = 0.$$

The optimal solution is $\frac{x_1+\delta(x_1+x_2)-(1-\lambda)\beta(x_1-x_2)^2}{2\lambda\beta(x_1-x_2)^2} = \frac{x_1+\delta(x_1+x_2)}{2\lambda\beta(x_1-x_2)^2} - \frac{1-\lambda}{2\lambda}$. For menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$ with $x_1 < x_2$, the utility for an interior choice p is as follows:

$$\lambda\left(0.5\left(1+p\right)x_1 + 0.5\delta\left(1+p\right)\left(x_1 + x_2\right) - \beta\left(0.5\left(1+p\right)\left(x_1 - x_2\right)\right)^2\right) \\ + (1-\lambda)\begin{pmatrix} p\left(x_1 + \delta\left(x_1 + x_2\right) - \beta\left(x_1 - x_2\right)^2\right) \\ + (1-p)\left(0.5x_1 + 0.5\delta\left(x_1 + x_2\right) - \beta\left(0.5\left(x_1 - x_2\right)\right)^2\right) \end{pmatrix}$$

and we obtain the following FOC:

$$\lambda\left(0.5x_1 + 0.5\delta\left(x_1 + x_2\right) - 0.5\beta\left(1+p\right)\left(x_1 - x_2\right)^2\right) \\ + (1-\lambda)\left(0.5x_1 + 0.5\delta\left(x_1 + x_2\right) - \frac{3}{4}\beta\left(x_1 - x_2\right)^2\right) = 0.$$

The optimal solution is given by: $\frac{x_1+\delta(x_1+x_2)}{\lambda\beta(x_1-x_2)^2} - \frac{3+\lambda}{4\lambda}$. The optimal chosen probabilities may lie in the interior or the corner, depending on parameter values. It is possible to have $\frac{x_1+\delta(x_1+x_2)}{\lambda\beta(x_1-x_2)^2} - \frac{3+\lambda}{4\lambda} > \frac{x_1+\delta(x_1+x_2)}{2\lambda\beta(x_1-x_2)^2} - \frac{1-\lambda}{2\lambda}$ given $\frac{x_1+\delta(x_1+x_2)}{2\beta(x_1-x_2)^2} > \frac{1+3\lambda}{4}$. If we have $\frac{x_1+\delta(x_1+x_2)}{\lambda\beta(x_1-x_2)^2} - \frac{3+\lambda}{4\lambda} > 0 > \frac{x_1+\delta(x_1+x_2)}{2\lambda\beta(x_1-x_2)^2} - \frac{1-\lambda}{2\lambda}$, a subject chooses $p = 0$ in menu $\begin{pmatrix} x_1, & x_2 \\ 0, & 0 \end{pmatrix}$ and an interior probability in menu $\begin{pmatrix} x_1, & x_2 \\ 0.5x_1, & 0.5x_2 \end{pmatrix}$. Moreover, we have optimal chosen probabilities decreasing from the upper to the lower part of the triangle if $\alpha < \beta$. Thus, this behavioral model could be compatible with the observed behavior.

Appendix 4: Experimental instructions and decision sheets

Experimental Instructions

In this decision making experiment, the task involves a pair of participants, **yourself** and **another participant** in this room. You will make decisions regarding possible payment for both of you as shown the decision table example below. Note that the numbers here are for illustrative purpose only.

Allocation 1		Allocation 2	
You	Other	You	Other
7	8	10	0
p:		***1-p:***	

Example 1 In this decision, there are two allocations: Allocation 1, you get \$7 and the other participant gets \$8; Allocation 2, you get \$10, and the other participant gets \$0. You are asked to choose a probability ***o*** to implement Allocation 1 and ***1-p*** to implement Allocation 2. You can choose any probability including 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.

Allocation 1		Allocation 2	
You	Other	You	Other
3	5	5	3
p:		***1-p:***	

Example 2 In this decision, there are two allocations: Allocation 1, you get \$3 and the other participant gets \$5; Allocation 2, you get \$5, and the other participant gets \$3. You are asked to choose a probability ***p*** to implement Allocation 1 and ***1-p*** to implement Allocation 2. You can choose any probability including 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.
You will make a number of choices similar to the examples. There is neither correct nor wrong answer to the tasks, and you choose your preferred probability for each of the decision tables. At the end of the experiment, we will randomly choose one participant to implement one of his or her choices, and match him/her to the other participant in this room.

The probability will be implemented by drawing one card from a set of 10 cards numbered from 1 to 10. If you choose ***p*** to be 0, Allocation 1 will be not implemented and Allocation 2 will be implemented regardless of the card you draw. If you choose ***p*** to be 0.1, Allocation 1 will be implemented if you draw number 1, otherwise Allocation 2 will be implemented. If you choose ***p*** to be 0.2, Allocation 1 will be implemented if you draw number 1 or 2, otherwise Allocation 2 will be implemented. If you choose ***p*** to be 0.3, Allocation 1 will be implemented if you draw number 1, 2, or 3, otherwise Allocation 2 will be implemented. And so on.

Exercise

While calculating payoffs seems easy, it is important that everyone understands. So, below we ask you to calculate the payoffs of both players for some specific examples. After you finish, we will go over the correct answers together.

Exercise 1.

Allocation 1		Allocation 2	
You	Other	You	Other
7	8	10	0
p: 0.4		***1-p:***0.6	

Suppose the table above is chosen for implementation, and you choose 0.4 for Allocation 1 and 0.6 for Allocation 2.

At the end of the experiment, a card is randomly drawn from a set of 10 cards numbered from 1 to 10.

If the card drawn is 3, your payment will be ____, and the payment of the other participant will be ____.

If the card drawn is 9, your payment will be ____, and the payment of the other participant will be ____.

Exercise 2.

Allocation 1		Allocation 2	
You	Other	You	Other
3	5	5	3
p: 1		***1-p:***0	

Suppose the table above is chosen for implementation, and you choose 1 for Allocation 1 and 0 for Allocation 2.

At the end of the experiment, a card is randomly drawn from a set of 10 cards numbered from 1 to 10.

If the card drawn is 3, your payment will be ____, and the payment of the other participant will be ____.

If the card drawn is 9, your payment will be ____, and the payment of the other participant will be ____.

This is the end of the instruction. Should you have any question, please raise your hand.

Sample Decision Sheet

Allocation 1		Allocation 2	
You	Other	You	Other
0	20	0	0
p:		***1-p:***	

Other decision sheets are presented in a similar manner. The decision sheets are randomly presented to each subject

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