# Partial Ambiguity

CHEW Soo Hong, MIAO Bin, and ZHONG Songfa*

November 2016

**Abstract**

We extend Ellsberg's two-urn paradox and propose three symmetric forms of partial ambiguity by limiting the possible compositions in a deck of 100 red and black cards in three ways. Interval ambiguity involves a symmetric range of $50-n$ to $50+n$ red cards. Complementarily, disjoint ambiguity arises from two nonintersecting intervals of 0 to $n$ and $100-n$ to 100 red cards. Two-point ambiguity involves $n$ or $100-n$ red cards. We investigate experimentally attitudes towards partial ambiguity and the corresponding compound lotteries in which the possible compositions are drawn with equal objective probabilities. This yields three key findings: distinct attitudes towards the three forms of partial ambiguity, significant association across attitudes towards partial ambiguity and compound risk, and source preference between two-point ambiguity and two-point compound risk. Our findings help discriminate among models of ambiguity in the literature.

Keywords: Risk, Ambiguity, Ellsberg paradox, Experiment, Choquet expected utility, Maxmin expected utility, Recursive non-expected utility, Source preference.

---

*We would like to acknowledge Yi-Chun Chen, Lorenz Goette, Yoram Halevy, John Ham, Zhihua Li, Yunfeng Lu, Xiangyu Qu, Jacob Sagi, Changcheng Song, Peter Wakker, audiences at the conferences of Foundations of Utility and Risk (2012), Risk, Uncertainty and Decision (2013), Decision: Theory, Experiments, and Applications (2014) and seminar participants at Caltech, Erasmus University, George Mason University, University of California San Diego, University of Hong Kong, McGill University, Vanderbilt University, Wuhan University, and Xiamen University, for their helpful comments. The paper has benefited significantly from the comments of a number of anonymous referees. We would like to thank Foo See Zuo for providing excellent research assistance. For financial supports, Chew and Zhong thank the National University of Singapore, and Ministry of Education, Singapore, and Miao acknowledges the China National Science Foundation (Grant 71503158). The paper combines two early working papers, namely "Partial Ambiguity" by Chew, Miao and Zhong, and "An Experimental Study of Attitude towards Compound Lottery" by Miao and Zhong. Chew: Department of Economics, National University of Singapore, chewsoohong@gmail.com. Miao: School of Economics, Shanghai University of Finance and Economics, binmiao11@gmail.com, Zhong: Department of Economics, National University of Singapore, zhongsongfa@gmail.com.

# 1 Introduction

In his 1921 classic, Risk, Uncertainty and Profit, Knight distinguishes between *risk*, where probabilities are measurable and known, and *uncertainty*, where probabilities are unmeasurable and unknown. He further offers his view on the essence of decision making under uncertainty as having to do with "action according to opinion, of greater or less foundation and value, neither entire ignorance nor complete and perfect information, but partial knowledge". In the same year, Keynes, in his Treatise on Probability, draws a distinction between probability and the body of knowledge or degree of confidence in assessing it. To illustrate this point, Keynes shares a thought experiment that has reappeared independently as Ellsberg's (1961) two-urn paradox. Between betting on an urn with a known composition of 50 red and 50 black balls over betting on another urn with an unknown composition of red and black balls which sum to 100, people may prefer to bet on the former known urn over the latter unknown urn. Such behavior, labeled as ambiguity aversion, casts doubt on the notion of subjective probability as a foundation for decision making under uncertainty.

The phenomenon of aversion to ambiguity has given rise to a voluminous literature from which we can discern three perspectives. An early perspective, traceable to Ellsberg (1961), views ambiguity as encompassing a range of possible priors from the different compositions stemming from the unknown urn. Ambiguity aversion may then arise from a sense of pessimism in disproportionately weighting the less desirable priors. This *multiple-prior perspective* is reflected in the Choquet expected utility model and maxmin expected utility.[1] A second perspective arises from viewing an ambiguous lottery as a two-stage (compound) lottery with a subjective symmetric probability distribution at the first stage over the possible priors from different compositions at the second stage (see, e.g., Becker and Brownson, 1964; Segal, 1987). Ambiguity aversion then corresponds to a compound lottery being less preferred than the even-chance lottery derived from it through reduction. This behavior violates the reduction of compound lottery axiom (RCLA) in which a compound lottery is indifferent to its reduction to a simple lottery. Recent models of ambiguity attitude adopting such a *two-stage perspective*, but not requiring RCLA, include recursive expected utility and recursive rank-dependent utility.[2] A third perspective, attributable to Keynes and revisited in Fox and Tversky (1995), views the complementary events—red and black—from different urns as uncertainties arising from distinct sources. In models of ambiguity attitude adopting this

[1] These include Schmeidler (1989) and Gilboa (1987) on Choquet expected utility and subsequently their joint work on maxmin expected utility (Gilboa and Schmeidler, 1989).

[2] Recursive expected utility has appeared in the works of Klibanoff, Marinacci and Mukerji (2005) and Seo (2009). Recursive rank-dependent utility has earlier appeared in Segal (1987, 1990).

*source perspective*,[3] ambiguity aversion corresponds to an even-chance bet on the known urn being preferred to an even-chance bet on the unknown urn with the same outcomes.

As espoused by Knight, the essence of decision making under uncertainty is partial knowledge that may arise from vague evidence, diverse information or conflicting news.[4] Going beyond pure risk and full ambiguity in the original two-urn paradox, this paper experimentally studies decision making in a richer domain of uncertainty involving intermediate forms of ambiguity, and analyzes the implications of our findings on models of ambiguity under the three perspectives. Specifically, we examine attitude towards three kinds of partial ambiguity using different decks of 100 cards that are individually either red or black. One kind of partial ambiguity, called interval ambiguity and denoted by $[50-n, 50+n]$, is based on Becker and Brownson (1964) and involves a symmetric range of possible compositions where the number of red or black cards lies between $50-n$ and $50+n$. Another kind of partial ambiguity, called disjoint ambiguity and denoted by $[0, n] \cup [100-n, 100]$, consists of a union of two nonintersecting intervals of possible compositions. Here, the number of red or black cards is either not more than $n$ or not less than $100-n$. The third kind of partial ambiguity, called two-point ambiguity and denoted by $\{50-n, 50+n\}$, involves precisely two possible compositions—either $50-n$ red cards or $50+n$ red cards with the rest of the cards black. In our experiment, we vary $n$ to obtain different decks of cards with varying level of knowledge on their compositions. This generates different partial ambiguity lotteries through bets on the color of a card drawn from each of these decks. For both interval and disjoint ambiguity, the number of possible compositions increases in $n$ but this remains the same for two-point ambiguity whose spread increases in $n$ instead. The structure of the three kinds of partial ambiguity is illustrated in Figure 1 in the next section where we elaborate further on our experimental design.

To fully investigate attitude towards partial ambiguity and the implications on different models, our experiment includes the corresponding compound lottery for each kind of partial ambiguity. We construct each compound lottery by inducing a uniform distribution on the possible compositions so that they are objectively equally likely and the compound lotteries all reduce to the same even-chance lottery. The inclusion of compound lotteries builds on Halevy's (2007) extension of the original Ellsberg experiment, in which he observes a close link between attitudes towards full ambiguity and whether subjects exhibit RCLA. We implement our experiment using both within-subject and between-subject designs on partial

[3]Source preference has been axiomatized in Nau (2006), Chew and Sagi (2008) and Ergin and Gul (2009).

[4]Ordinarily, analysts' views on market performance tend to coalesce around some interval. Yet, there are times when such views and news reports turn out to be conflicting and polarizing, e.g., announcements and reports from different sources during the volatile phase around the plunge of China's stock markets in July and August, 2015 (Bloomberg Business, August 25, 2015).

ambiguity and compound lottery (see Section 2 for further details). We examine partial ambiguity attitude together with the corresponding compound risk attitude (compound risk and compound lottery are used interchangeably). In particular, for each lottery, we elicit its certainty equivalent using a price-list mechanism and infer attitudes towards partial ambiguity (compound risk) by comparing the certainty equivalents of partial ambiguity (compound risk) with that of the even-chance lottery.

We summarize the predictions for models of ambiguity under the three perspectives. Multiple-prior models such as Choquet expected utility and maxmin expected utility tend to enjoy considerable flexibility in generating various predictions on attitude towards the three forms of partial ambiguity. In adopting the Anscombe-Aumann (1963) framework, these two models satisfy RCLA and reduce all the compound lotteries in our experiment to the simple even-chance lottery. It follows that these models predict that there is no direct link between attitude towards ambiguity and attitude towards compound risk. By contrast, in relaxing RCLA, models under the two-stage perspective, including recursive expected utility and recursive rank-dependent utility, predict similar choice patterns across the domains of partial ambiguity and compound risk, as well as correlation in attitudes between the two domains.

Both recursive expected utility and recursive rank-dependent utility can generate distinct choice behavior in the three forms of partial ambiguity and the corresponding compound risks. In particular, an ambiguity averse decision maker applying the same rank-dependent utility recursively will exhibit aversion towards increasing the number of possible compositions for both interval and disjoint ambiguity as well as compound risk. Meanwhile, an ambiguity averse decision maker with recursive expected utility preference will exhibit opposing attitudes towards increasing the number of possible compositions—aversion to increasing the number of possible compositions for interval ambiguity and compound risk and aversion to decreasing the number of possible compositions for disjoint ambiguity and compound risk. For two-point ambiguity and compound risk, an ambiguity averse decision maker with recursive expected utility exhibits aversion towards increasing spread while a decision maker with recursive rank-dependent utility preference exhibits aversion towards increasing spread except near the end point $\{0, 100\}$.

In encompassing both recursive rank-dependent utility and recursive expected utility as special cases, the source preference model of Ergin and Gul (2009) can exhibit the aforementioned predictions of either model for the three forms of partial ambiguity and the corresponding compound risks. Between two-point ambiguity and two-point compound risk, the Ergin-Gul model along with both recursive expected utility and recursive rank-dependent utility predicts indifference under a symmetry assumption in which the two possible compositions are subjectively equally likely.

In relation to these predictions, we have three key findings:

*Key Finding 1*: Aversion to increasing the number of possible compositions for interval and disjoint ambiguity (compound risk), and aversion to increasing spread in two-point ambiguity (compound risk) except near the end-point.

*Key Finding 2*: Significant association between attitudes towards partial ambiguity and compound risk.

*Key Finding 3*: Non-indifference between two-point ambiguity and the corresponding two-point compound risk.

Together, these findings reveal a rich range of choice behavior which can help distinguish among various models of ambiguity under the three perspectives. In focusing on within-domain choice behavior, Key Finding 1 nevertheless reveals a strong similarity in attitudes towards ambiguity and compound risk. This corroborates Key Finding 2 of a significant correlation between attitudes towards partial ambiguity and compound risk which extends Halevy's (2007) original finding from full ambiguity to partial ambiguity. The overall link between ambiguity attitude and compound risk attitude is consistent with both two-stage models but not with models of ambiguity satisfying RCLA. The shared aversion to the number of possible compositions for interval and disjoint ambiguity and compound risk in Key Finding 1 discriminates between the two-stage models in favor of recursive rank-dependent utility over recursive expected utility. Comparing across two-point ambiguity and compound risk, Key Finding 3 points to a differentiation between subjective and objective even-chance priors at the first stage. This finding is not compatible with the prediction of indifference for the Ergin-Gul model, encompassing both recursive expected utility and recursive rank-dependent utility, and underscores the value of incorporating source preference for further theoretical development.

# 2 Experimental Design

This section presents the framework for our experimental study comprising a main experiment and two supplementary experiments (S1 and S2). The main experiment is a within-subject study of partial ambiguity and compound risk jointly. The supplementary experiments, focusing on partial ambiguity and compound risk individually, enable us to examine the robustness of the results from the main experiment.

## 2.1 Experimental Framework

In an Ellsberg setting, let $\{50\}$ denote the known deck of 100 cards with 50 red cards and 50 black cards, and let $[0, 100]^A$ denote the unknown deck whose composition of the cards is unknown. We consider three intermediate forms of symmetric partial ambiguity (see Figure 1). Interval ambiguity, denoted by $\mathbf{I}_n^A = [50-n, 50+n]^A$, refers to a deck containing between $50-n$ and $50+n$ red (black) cards. Disjoint ambiguity, denoted by $\mathbf{D}_n^A = [0, n] \cup [100-n, 100]^A$, refers to a deck whose number of red (black) cards is either between 0 and $n$ or between $100-n$ and 100. Two-point ambiguity, denoted by $\mathbf{T}_n^A = \{50-n, 50+n\}^A$, refers to a deck containing either $50-n$ or $50+n$ red (black) cards.

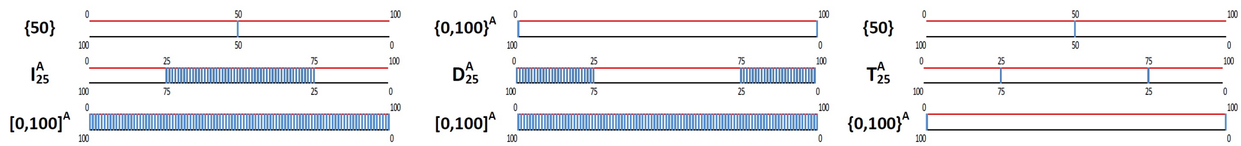



Figure 1: Illustration of three forms of partial ambiguity. *The upper line represents the number of red cards and the lower line represents the number of black cards, while each vertical line represents one possible composition of the deck.*

To further examine the potential link between ambiguity and compound risk, we include three kinds of compound risk corresponding to the three kinds of partial ambiguity: interval compound risk denoted by $\mathbf{I}_n^C = [50-n, 50+n]^C$, disjoint compound risk denoted by $\mathbf{D}_n^C = [0, n] \cup [100-n, 100]^C$ and two-point compound risk denoted by $\mathbf{T}_n^C = \{50-n, 50+n\}^C$. Here, compound risk is implemented with objective uniform stage-1 priors, i.e., each possible composition is equally likely.

We use the notation for decks of cards to also denote lotteries generated by betting on the color of a card drawn from the corresponding deck. Note that the bets associated with the three kinds of partial ambiguity and compound risk are symmetric in the sense that betting on either color results in the same description of the possible compositions. Therefore, we do not distinguish between the colors that the bets are placed on.

Our design enables us to examine the effect of varying the number of possible compositions in both interval and disjoint ambiguity and the corresponding interval and disjoint compound risk, as well as the effect of varying the spread in two-point ambiguity and compound risk. In addition, our design enables us to examine the extent of correlation between attitude towards partial ambiguity and attitude towards compound risk, and whether subjects have distinct attitudes between partial ambiguity and compound risk.

## 2.2 Main Experiment

This subsection presents the detailed design of our main experiment which combines both ambiguity and compound lotteries in a within-subject setting. The experiment considers the following three kinds of partial ambiguity lotteries.

*Interval ambiguity*: $\{50\}$, $\mathbf{I}_{25}^{A} = [25, 75]^{A}$, and $[0, 100]^{A}$.

*Disjoint ambiguity*: $\{0, 100\}^{A}$, $\mathbf{D}_{25}^{A} = [0, 25] \cup [75, 100]^{A}$, and $[0, 100]^{A}$.

*Two-point ambiguity*: $\{50\}$, $\mathbf{T}_{25}^{A} = \{25, 75\}^{A}$, and $\{0, 100\}^{A}$.

Correspondingly, we consider three kinds of uniform compound lotteries.

*Interval compound risk*: $\{50\}$, $\mathbf{I}_{25}^{C} = [25, 75]^{C}$, and $[0, 100]^{C}$.

*Disjoint compound risk*: $\{0, 100\}^{C}$, $\mathbf{D}_{25}^{C} = [0, 25] \cup [75, 100]^{C}$, and $[0, 100]^{C}$.

*Two-point compound risk*: $\{50\}$, $\mathbf{T}_{25}^{C} = \{25, 75\}^{C}$, and $\{0, 100\}^{C}$.

As $\{50\}$ is shared by partial ambiguity lotteries and compound lotteries, the experiment consists of 11 lotteries. Prior to the experiment, we construct the 50-50 deck for known risk, five decks of cards for the ambiguity lotteries, and five bags containing numbered tickets for the compound lotteries. For each partial ambiguity lottery, we construct the decks of cards following the description of the range of possible compositions. For example, to construct interval ambiguity $\mathbf{I}_{25}^{A}$, we select 50 cards with completely unknown composition together with 25 red cards and 25 black cards. For each compound lottery, we implement its objective uniform stage-1 prior as follows: one ticket is drawn randomly from a bag containing numbered tickets, and the number drawn determines the number of red cards in the deck with the rest black. Subjects bet on the color of a card drawn from the deck before knowing the number drawn. We present the 11 lotteries using 11 corresponding slides and display the actual decks of cards and bags of numbered tickets to the subjects at the same time. For each lottery, a subject chooses the color of a card to bet on; the subject receives SGD40 (about USD30) if the bet is correct, and receive nothing otherwise. Letting subjects choose the color of the card ameliorates potential effects of suspicion without removing the possible incidence of color preference.

To elicit the certainty equivalent (henceforth CE) of each lottery, we use a price-list design, with subjects choosing between a given lottery and a sure amount from a list of 21 amounts ranging from SGD8 to SGD32 with the expected payoff of SGD20 in the middle.[5] In general, subjects will choose the lottery when its CE is higher than the sure amount, and switch to the sure amount as it increases to the point that is higher than the CE for the lottery. Thus the switch point serves as a proxy for CE of the lottery. Subjects make 21 binary choices for

[5] While middle outcome could bias the absolute magnitude of the elicited CEs, it is unlikely to bias the relative magnitudes of the elicited CEs from which we infer the preference ordering across lotteries.

each lottery, resulting in 231 choices in total for 11 lotteries. To incentivize participation, in addition to a SGD5 show-up fee, we adopt the random incentive mechanism, paying each subject based on one of her randomly selected decisions in the experiment.[6]

The experiment is conducted in two different orders between ambiguity appearing first and compound risk appearing first. In the ambiguity first treatment, subjects make choices in 6 decision tables comprising $\{50\}$ and 5 ambiguity lotteries followed by choices for the 5 compound lotteries. In the compound risk first treatment, subjects make choices in 6 decision tables comprising $\{50\}$ and 5 compound lotteries followed by choices for the 5 ambiguity lotteries. Within each treatment, the order of presentation of the lotteries is randomized.

We have recruited 188 undergraduate students from the National University of Singapore (NUS) as participants by advertising on its Integrated Virtual Learning Environment. The experiment consists of 8 sessions with 20 to 30 subjects for each session. Altogether, 102 subjects are allocated to the ambiguity first treatment, and 86 subjects are allocated to the compound risk first treatment. Upon arriving at the experimental venue, subjects are given the consent form approved by the NUS Institutional Review Board. Subsequently, general instructions are read to the subjects followed by a demonstration of several examples of possible compositions of the deck before subjects begin making decisions (see Appendix E for Experimental Instructions). Most subjects complete the decision making tasks within 40 minutes. The payment stage takes about 40 minutes.

## 2.3 Supplementary Experiments

We briefly describe two supplementary experiments that serve as robustness checks of the findings from our main experiment. Both experiments include a wider range of $n$ compared with the main experiment. Experiment S1 investigates attitude towards the three kinds of partial ambiguity individually: $\mathbf{I}_n^A = [50-n, 50+n]^A$, $\mathbf{D}_n^A = [0, n] \cup [100-n, 100]^A$, and $\mathbf{T}_n^A = \{50-n, 50+n\}^A$ with $n$ taking on the values of 0, 10, 20, 30, 40, and 50. Experiment S2 investigates attitude towards the three kinds of compound lotteries individually: $\mathbf{I}_n^C = [50-n, 50+n]^C$, $\mathbf{D}_n^C = [0, n] \cup [100-n, 100]^C$, and $\mathbf{T}_n^C = \{50-n, 50+n\}^C$ with $n$ taking on the values of 0, 20, 40, and 50. As with the main experiment, both S1 and S2 use price list and random incentive mechanism to elicit the CEs of the various lotteries. Appendix D details the experimental design and results for the supplementary experiments. Appendix E contains Experimental Instructions.

[6]The use of random incentive mechanisms to elicit valuations has given rise to debates from the perspectives of both theory and experiment (see, e.g., Baillon, Halevy and Li, 2014 for detailed discussions in the setup of ambiguity). We adopt this mechanism with price list as it is simple and enables analysis of choice behavior at the individual level. Alternatively, Halevy (2007) uses the Becker–DeGroot–Marschak (1964) mechanism and pays all four bets in his experiment.

# 3 Theoretical Predictions

Following the exposition of our experimental design, this section analyzes how various models of ambiguity predict individual choice in our setting of partial ambiguity and compound risk. Beginning with the benchmark subjective expected utility model (SEU), we discuss a number of ambiguity models under the three perspectives described in the Introduction. In particular, we focus on four representative models: Choquet expected utility (CEU) under the multiple-prior perspective; recursive expected utility (REU) and recursive rank-dependent utility (RRDU) under the two-stage perspective; and the source preference model in Ergin and Gul (2009). Additional models are discussed in more detail in Appendix A.

For a given set of lotteries, each lottery $L_i$ pays $w$ if the subject correctly guesses the color—red ($R$) or black ($B$)—of the card drawn. To facilitate our derivation of the predictions of different utility models on partial ambiguity and compound risk, we impose the following assumption of symmetry which we will discuss further in Section 5.

*Symmetry*: For each $L_i$, the decision maker is indifferent between betting on $R$ or $B$.

Under the benchmark SEU model or more generally probabilistic sophistication (Machina and Schmeidler, 1992), the probabilities of the events $R$ and $B$ always equal 0.5, given symmetry. In particular,

$$U_{SEU}(L_i) = u\,(w)\,/2,$$

where we normalize $u\,(0) = 0$. Thus, SEU predicts that all ambiguity and compound lotteries in our setting and $\{50\}$ have the same CEs.

## 3.1 Multiple-prior Models

As observed in Ellsberg (1961), ambiguity arises from a choice situation involving a range of possible priors. This perspective has led to two important models of decision making under ambiguity, namely, CEU and maxmin expected utility (MEU) (Gilboa and Schmeidler, 1989). Here, we focus our discussion on the predictions of CEU since MEU can be largely accommodated by CEU in our setup as discussed below.

*Choquet expected utility.* As axiomatized in Gilboa (1987) and Schmeidler (1989), CEU generalizes SEU with a capacity function $\nu$, a non-additive extension of a probability measure that maps events into the unit interval and is monotonic in terms of inclusion. Under CEU, the utility for an ambiguous lottery $L_i^A$ is given below. Since the utilities of betting on either color are the same given symmetry, we use only the utility of betting on $R_{L_i}$ in our subsequent exposition.

$$U_{CEU}(L_i^A) = \nu(R_{L_i^A})u\,(w)\,,$$

where $R_{L_i^A}$ refers to the event of drawing red in $L_i^A$, with $\nu(R_{L_i^A}) = \nu(B_{L_i^A})$ from symmetry.[7] In relaxing additivity, the capacities or decision weights assigned to $R$ (or $B$) for different ambiguity lotteries need not be the same. Thus, CEU exhibits considerable flexibility in being compatible with a wide range of choice behavior, including both ambiguity aversion and ambiguity tolerance, without further restrictions on $\nu$. For the case of a convex $\nu$, i.e., $\nu(R) + \nu(B) \leq \nu(R \cup B) + \nu(R \cap B)$, CEU exhibits global ambiguity aversion, i.e., $\{50\}$ is preferred to any ambiguity lottery.

The MEU model evaluates an ambiguous lottery $L_i^A$ in terms of the expected utility of the worst prior in a convex set of priors $\Pi_{L_i^A}$. In our setup, the MEU model coincides with the CEU model with a convex capacity, and thus shares the same predictions including global ambiguity aversion. In the domain of compound risk, the CEU model axiomatized in Schmeidler (1989) predicts the same CEs for all compound lotteries in our setting since it adopts the Anscombe-Aumann framework which incorporates RCLA. In contrast, the axiomatization of CEU in Gilboa (1987) and Wakker (1987) in a Savagian domain is silent on how compound lotteries may be evaluated. These observations also apply to MEU models including those adopting the Anscombe-Aumann framework (e.g., Gilboa and Schmeidler, 1989) and those adopting a Savagian domain (Casadesus-Masanell, Klibanoff and Ozdenoren, 2000; Alon and Schmeidler, 2014). Appendix A provides a detailed analysis of the predictions of MEU and a number of follow-up models adopting the multiple-prior perspective in the domain of partial ambiguity.

## 3.2 Two-stage Models

Under the two-stage approach, an ambiguous lottery is associated with a stage-1 prior $\pi$, a distribution on a range of stage-2 priors derived from the set of possible compositions. Each stage-2 prior $\mu$ corresponds to a simple lottery that pays $w$ with $\mu(R)$ (or $\mu(B)$, depending on the color chosen) and 0 otherwise. It follows that $\pi$ induces a subjective compound lottery to be evaluated in two steps. For each stage-2 simple lottery induced by $\mu$, we first assess its CE as $c_\mu$. As is done in Segal (1987), the overall lottery is then evaluated as a stage-1 simple lottery $(\pi(\mu), c_\mu; \pi(\mu'), c_{\mu'}; ...)$, induced by the stage-1 prior and pays each $c_\mu$ with probability $\pi(\mu)$. Note that being indifferent to betting on either color regardless of the underlying two-stage preference implies symmetric stage-1 priors about $\{50\}$.[8] The following

[7] In Fox and Tversky's (1998) CEU model, the capacity takes the form of $f \circ P$, where $f$ is a probability weighting function and $P$ is a possibly non-additive judged probability.

[8] Otherwise, should there be a pair of symmetrically placed stage-2 risks, $\{n\}$ and $\{100-n\}$, whose assigned stage-1 probabilities are different, then we can pick a monotone stage-1 utility which is responsive to the difference between the CEs for $\{n\}$ and $\{100-n\}$ while minimizing the contributions of all other pairs. It follows that the overall indifference can no longer hold when the probabilities assigned to $\{n\}$ and $\{100-n\}$ along with those for the other pairs become reversed when switching from betting on red to betting on black.

definition of *stage-1 simple spread* (Machina, 1982) is based on a weakening of the standard definition of mean-preserving spread in risk using a single-crossing property.

**Definition**: For given symmetric stage-1 distributions $F$ and $G$ on $\mu$, we say $G$ is a *stage-1 simple spread* of $F$ if (i) $F(\mu) \leq G(\mu)$ for $\mu(R) < 0.5$; (ii) $F(\mu) \geq G(\mu)$ for $\mu(R) > 0.5$; and (iii) $F$ and $G$ reduce to the same simple lottery.

Several papers consider different recursive preference specifications in assessing $c_\mu$ and the induced stage-1 simple lottery. These papers include recursive expected utility (REU) involving distinct expected utilities in the two stages (Klibanoff, Marinacci and Mukerji, 2005; Seo, 2009),[9] and recursive rank-dependent utility (RRDU) involving the same rank-dependent utility in both stages (Segal, 1987; 1990). Halevy and Feltkamp (2005) link ambiguity aversion with aversion to mean-preserving spreads in an environment of bundled risks and deliver similar predictions as those of REU but not RRDU.

Note that two-stage models do not distinguish between objective and subjective stage-1 priors.[10] Therefore, both REU and RRDU deliver identical predictions for uniform compound lottery and partial ambiguity lottery under the assumption of uniform subjective stage-1 prior. In the sequel, we shall first focus on the case of uniform stage-1 priors without distinguishing whether the priors are subjective or objective, and subsequently discuss the predictions for general symmetric priors for partial ambiguity lotteries as well as predictions relating to objective uniform stage-1 priors.

*Recursive expected utility.* Given stage-1 utility function $u$ and stage-2 utility function $v$, the REU specification is given by:

$$U_{REU}(L_i) = \int u(c_\mu)\, dF, \text{ with } c_\mu = v^{-1}(v(w)\mu(R)),$$

where $F$ is the cumulative distribution function of $\pi$. Naturally, REU reduces to SEU if the stage-1 and stage-2 utility functions $u$ and $v$ are the same. Define a stage-1 relative curvature function as $u \circ v^{-1}$. Klibanoff, Marinacci and Mukerji (2005) show that REU can account for ambiguity aversion given that $u \circ v^{-1}$ is concave. With this concavity condition, we derive the predictions of REU below.

Given uniform stage-1 priors, we have specific stage-1 simple spread patterns in each kind of partial ambiguity and compound risk. In particular, we have stage-1 simple spread in two-point and interval lotteries as $n$ increases. In contrast, for disjoint lottery, we have stage-1 simple spread as $n$ decreases. As REU permits stage-1 expected utility, it predicts consistent aversion towards stage-1 simple spread when $u \circ v^{-1}$ is concave. It follows that

[9] Kreps and Porteus (1978) first propose and axiomatize REU in an intertemporal setting.

[10] One exception is Klibanoff, Mukerji and Marinacci's (2005) axiomatization of the REU model which adopts a Savagian domain and makes no predictions on its behavior in the domain of compound risks.

REU exhibits the following predictions which apply to both partial ambiguity and compound risk.

**Prediction 1**. Aversion (Affinity) to increasing $n$ in interval (disjoint) lotteries: $\mathbf{I}_n \succ \mathbf{I}_m$ and $\mathbf{D}_n \prec \mathbf{D}_m$ if $n < m$.

**Prediction 2.** Aversion to increasing $n$ in two-point lotteries: $\mathbf{T}_n \succ \mathbf{T}_m$ if $n < m$.

*Recursive rank-dependent utility.* Segal (1987, 1990) considers a two-stage RRDU specification with the same rank-dependent utility (Quiggin, 1982) in both stages:

$$U_{RRDU}(L_i) = \int u\left(c_\mu\right) df\left(F\right), \text{ with } c_\mu = u^{-1}\left(u\left(w\right) f\left(\mu\left(R\right)\right)\right),$$

where $f$ is a probability weighting function and $u$ a common utility function applied to both stages. Segal (1987) shows that RRDU can account for ambiguity aversion when $f$ is convex.

We provide some intuition here for the predictions of RRDU with a convex probability weighting function on each kind of partial ambiguity and compound risk, and relegate details of their proofs to Appendix A. For the case of interval lottery $\mathbf{I}_n$, as $n$ increases, stage-1 decision weight on the best stage-2 lottery, $\{50 + n\}$, becomes disproportionately small while stage-1 decision weight on the worst stage-2 lottery, $\{50 - n\}$, becomes disproportionately large. The overall effect of the change in decision weights more than offsets the effect of increasing the utility of $\{50 + n\}$ relative to that of $\{50 - n\}$. This gives rise to an overall aversion to increasing $n$ in interval lottery. A similar intuition being applied to disjoint lottery leads to the following predictions of RRDU for both partial ambiguity and compound risk.

**Prediction 3**. Aversion to increasing $n$ in interval and disjoint lotteries: $\mathbf{I}_n \succ \mathbf{I}_m$ and $\mathbf{D}_n \succ \mathbf{D}_m$ if $n < m$.

For two-point lotteries, after normalizing $u\left(w\right) = 1$, we have

$$U_{RRDU}(\mathbf{T}_n) = \left(1 - f\left(0.5\right)\right) f\left(0.5 - \widehat{n}\right) + f\left(0.5\right) f\left(0.5 + \widehat{n}\right),$$

where $\widehat{n} = n/100$ with $n$ varying between 0 and 50. A reversal in two-point lotteries follows from observing that the above expression is convex in $\widehat{n}$ and admits the same value at $\widehat{n} = 0$ and 0.5. This yields the following.

**Prediction 4.** Initial aversion to increasing $n$ for two-point lottery followed by a reversal: there exists an $n^*$ such that $\mathbf{T}_n \succ \mathbf{T}_m$ for $n < m \leq n^*$, $\mathbf{T}_m \succ \mathbf{T}_n$ for $n^* \leq n < m$, and $\{50\} \sim \{0, 100\}$.

Besides REU and RRDU, we can consider other preference specifications for a recursive model, e.g., betweenness utility (Chew, 1983; Dekel, 1986) and quadratic utility (Chew, Epstein and Segal, 1991). Recursive betweenness utility can exhibit aversion to stage-1 simple spreads under certain conditions as characterized in Chew (1989) and Dekel (1986), such that it can generate the same predictions as those of REU. Recursive quadratic utility can share similar predictions with RRDU since quadratic utility and rank-dependent utility have common intersections when the probability weighting function is itself quadratic (see Chew, Epstein and Segal (1991) for details).

*Non-uniform priors.* In the domain of partial ambiguity, uniform stage-1 prior is one possible symmetric prior underpinning a two-stage perspective. We next proceed to check the extent to which the predictions above still hold under weaker conditions of non-uniform but symmetric stage-1 priors. We first observe that Prediction 2 for REU and Prediction 4 for RRDU in the domain of partial ambiguity are robust to general symmetric priors given that they are necessarily even-chance in two-point ambiguity. In addition, since a concave relative curvature function in REU predicts aversion to all kinds of stage-1 simple spread, Prediction 1 for REU in the domain of partial ambiguity remains unchanged if the priors in interval and disjoint ambiguity satisfy the corresponding stage-1 simple spread condition, e.g., the stage-1 prior in $\mathbf{I}_m^A$ is a stage-1 simple spread of that in $\mathbf{I}_n^A$, and the stage-1 prior in $\mathbf{D}_n^A$ is a stage-1 simple spread of that in $\mathbf{D}_m^A$, for $n < m$.[11] With regard to Prediction 3 of RRDU in the domain of partial ambiguity, besides the requirement that stage-1 priors in interval and disjoint ambiguity satisfy the corresponding stage-1 simple spread conditions as that for REU, the robustness of Prediction 3 would rely on additional conditions. Specifically, the uniform prior in $[0, 100]^A$ is a stage-1 simple spread for all stage-1 priors in interval ambiguity and all stage-1 priors in disjoint ambiguity are stage-1 simple spreads of the uniform prior in $[0, 100]^A$.[12] Notably, under the condition of stage-1 simple spread, REU has clear predictions on choice behavior across the three kinds of partial ambiguity, i.e., $\mathbf{I}_n^A$ is preferred to $\mathbf{T}_n^A$, which is in turn preferred to $\mathbf{D}_{50-n}^A$. By contrast, we do not have such clear predictions for RRDU under the same stage-1 simple spread condition. This can be seen from observing that there is some $n'$ such that the CE of $\mathbf{T}_{n'}^A$ exceeds that of $\mathbf{I}_{n'}^A$ under uniform stage-1 prior in view of predictions 3 and 4. Yet, since the CE of $\{50\}$ exceeds that of $\mathbf{T}_{n'}^A$, there exists a binomial stage-1 prior in $\mathbf{I}_{n'}^A$, which is slightly perturbed from that in $\{50\}$, such that the CE of $\mathbf{I}_{n'}^A$ remains greater than that of $\mathbf{T}_{n'}^A$ by continuity.

*Focusing on compound risk.* In the domain of compound risk with objective uniform stage-1 priors, aversion to stage-1 simple spread entails an aversion to increasing the number of

[11]Besides a uniform prior, the stage-1 simple spread condition is compatible with a binomial belief (treating each card as being equally likely to be red or black), a geometric belief (viewing the unknown deck of 100 cards as a random draw from a deck of 100 red and 100 black cards), and a U-shaped belief which captures a more extreme view in which the worst and best outcomes are disproportionately more likely to occur.

[12]In Appendix A, we prove a more general result. Given two symmetric stage-1 priors $F$ and $G$ such that $G$ is a stage-1 simple spread of $F$, the compound lottery represented by $F$ is preferred to the compound lottery represented by $G$ if both $F$ and $G$ are stage-1 simple spreads of the uniform prior in $[0, 100]^A$. Conversely, we have $G \succ F$ if the uniform prior in $[0, 100]^A$ is a stage-1 simple spread of both $F$ and $G$.

possible compositions in interval compound risk and a preference for increasing the number of possible compositions in disjoint compound risk. In this regard, a REU decision maker with a concave relative curvature function exhibits aversion to stage-1 simple spread for both interval and disjoint compound risk, while a RRDU decision maker with a convex probability weighting function exhibits opposite attitudes towards stage-1 simple spread for these two cases. Moreover, our design involving objective uniform stage-1 priors enables further tests of specific axioms underlying REU and RRDU on decision making under compound risk. In our axiomatic analysis, besides RCLA, we examine a weaker axiom—*time neutrality*—which requires a compound lottery which completely resolves in stage-2 to be indifferent to another which resolves completely in stage-1 but reduces to the same simple lottery as the first lottery. Note that time neutrality holds when stage-1 and stage-2 preferences are the same, such as in RRDU. In our setup, time neutrality implies $\{50\} \sim \{0, 100\}^C$, since they differ only in terms of timing of uncertainty resolution with uncertainty resolving later in $\{50\}$ and resolving earlier in $\{0, 100\}^C$.[13] We can further test the predictions of different models on stage-1 preference by applying the independence and betweenness axioms to stage-1 prior over compound lotteries, giving rise to *stage-1 independence* and *stage-1 betweenness*. Notice that under our design of uniform stage-1 prior, $[0, 100]^C$ can be expressed as a stage-1 probability mixture of $\mathbf{I}_n^C$ and $\mathbf{D}_{50-n}^C$ assuming that the two overlapping points are negligible. It follows that a stage-1 preference satisfying independence or more generally, betweenness, such as in REU, yields a crisp and easily testable prediction that $[0, 100]^C$ would be intermediate in preference between $\mathbf{I}_n^C$ and $\mathbf{D}_{50-n}^C$.[14] Appendix B provides a detailed exposition of our axiomatic analysis.

### 3.3 Source Models

Building on the empirical findings of Fox and Tversky (1995), models incorporating source preference have been proposed in Nau (2006), Chew and Sagi (2008), and Ergin and Gul (2009). Chew and Sagi (2008) axiomatize source preference directly in terms of probabilistic sophistication on smaller families of events that they label as small worlds. This approach exhibits RCLA for risks arising from each small world as a source of uncertainty (see Appendix A for further analysis of the predictions of this model). Both Nau (2006) and Ergin and Gul (2009) incorporate the two-stage perspective while bringing in source considerations

[13] One concern is that subjects may differentiate $\{50\}$ from a degenerate compound lottery $(\{50\}, 1)$, which delivers $\{50\}$ with probability 1, mindful that time neutrality implies indifference between $(\{50\}, 1)$ and $\{0, 100\}^C$. In an experiment with 53 subjects, we elicit the CEs of $\{50\}$ and $(\{50\}, 1)$ using a between-subject design, and do not find any significant difference between the two.

[14] The assumption of stage-1 independence in Klibanoff, Marinacci and Mukerji (2005) is discussed in Epstein (2010) and subsequently in Klibanoff, Marinacci and Mukerji (2012).

to differentiate between stage-1 and stage-2 priors. Nau's (2006) approach gives rise to an alternative axiomatization of REU and can exhibit predictions 1 and 2.

We focus on Ergin and Gul's (2009) axiomatization of a more general representation of second-order probabilistic sophistication (SPS) that admits non-expected utility preferences in both stages and contains REU and RRDU as special cases. In transforming partial ambiguity into subjective compound lotteries, SPS requires probabilistic sophistication at each stage, and allows different compound lotteries to be non-indifferent even when they reduce to the same simple lottery. In incorporating rank-dependent utility (possibly distinct) for the two stages, SPS can exhibit predictions 3 and 4. Notably, SPS does not require indifference between $\{50\}$ and $\{0, 100\}$ since it can be more flexible than RRDU in accommodating distinct within-stage rank-dependent utility preferences across the two stages thereby relaxing the time neutrality axiom.

## 3.4 Linking Ambiguity with Compound Risk

The models above deliver various predictions in the respective domains of partial ambiguity and compound risk. In particular, models under the multiple-prior perspective and under two-stage perspective make different predictions when the association across the two domains is considered (Halevy, 2007). More specifically, multiple-prior models adopting the Anscombe-Aumann framework with RCLA predict indifference among the compound lotteries in our design, while those models axiomatized in a Savagian domain in which compound lottery is absent are silent on whether there is a link between ambiguity attitude and compound risk attitude. In contrast, under certain assumptions on stage-1 priors, models relaxing RCLA including REU, RRDU, and SPS predict similarity in choice behavior under partial ambiguity and compound risk as well as association between partial ambiguity attitude and compound risk attitude.

We note that under two-stage models, the CEs for ambiguity lotteries in general differ from those of the corresponding uniform compound lotteries except under subjective uniform stage-1 priors (Amarante, Halevy and Ozdenoren, 2011). For example, in REU, a unimodal prior such as binomial in $[0, 100]^A$ would deliver a higher CE than that for $[0, 100]^C$ while a U-shaped prior would imply the reverse. The corresponding prediction is distinct for RRDU with a convex probability weighting function given that a U-shaped prior can still deliver a higher CE since it can be viewed as a stage-1 simple spread of the uniform prior (see preceding analysis for non-uniform priors in two-stage models). In our experimental design, there is one case, namely two-point ambiguity, where the CEs of the ambiguity lotteries and the corresponding compound lotteries are the same for REU, RRDU, and the SPS model, under the assumption that the two possible compositions in two-point ambiguity are equally likely. Given that even-chance stage-1 priors arise subjectively in two-point ambiguity while they arise objectively in two-point compound risk, to have strict preference between these two types of two-point lotteries, one approach is to incorporate source preference between stage-1 priors arising subjectively and objectively in a model adopting a two-stage perspective.

# 4 Results

In this section, we first present the observed choice behavior for both partial ambiguity and compound risk in the main experiment. This is followed by discussions of the robustness of our observations in experiments S1 and S2, to be elaborated in greater detail in Appendix D. We then link attitudes towards partial ambiguity and compound risk through comparisons of their CEs and examining their correlations. We end the section with an individual type analysis in terms of the proximity of the observed behavior to the predictions of specific utility models.

## 4.1 Choice Patterns within Ambiguity and Compound Risk

Figure 2 summarizes the choice data, coded in terms of the switch point as proxies for the CE, separately for partial ambiguity and compound risk. Besides the standard findings of ambiguity aversion and compound risk aversion,[15] a striking similarity between attitude towards partial ambiguity and attitude towards compound risk is apparent from the plots of the overall choice patterns. Summarizing, we arrive at the following patterns of choice behavior for each of the three forms of partial ambiguity and compound risk.

**Observation 1A**: *For both interval ambiguity and compound risk, there is a decreasing trend in the CEs as the number of possible compositions increases.*

To test the relationship between CE and the number of possible compositions, we conduct a linear regression analysis with CE as the dependent variable and the number of possibilities as the independent variable with robust standard errors clustered at the individual level.[16]

[15]In particular, between $\{50\}$ and full ambiguity $[0, 100]^A$, 97 of the 188 subjects are ambiguity averse, 69 are ambiguity neutral, and 22 are ambiguity seeking. Between $\{50\}$ and $[0, 100]^C$, 100 are compound risk averse, 63 are compound risk neutral, and 25 are compound risk seeking. Aversion to ambiguity (compound risk) is similarly observed for the other 4 ambiguity (compound) lotteries, see Table C.I and Figure C.1 in Appendix C for details.

[16]We use Page's L trend as a robustness check. We find that aversion to the number of possibilities is robust for interval ambiguity ($p < 0.001$), interval compound risk ($p < 0.001$), disjoint ambiguity ($p < 0.014$), but not for disjoint compound risk ($p = 0.133$).

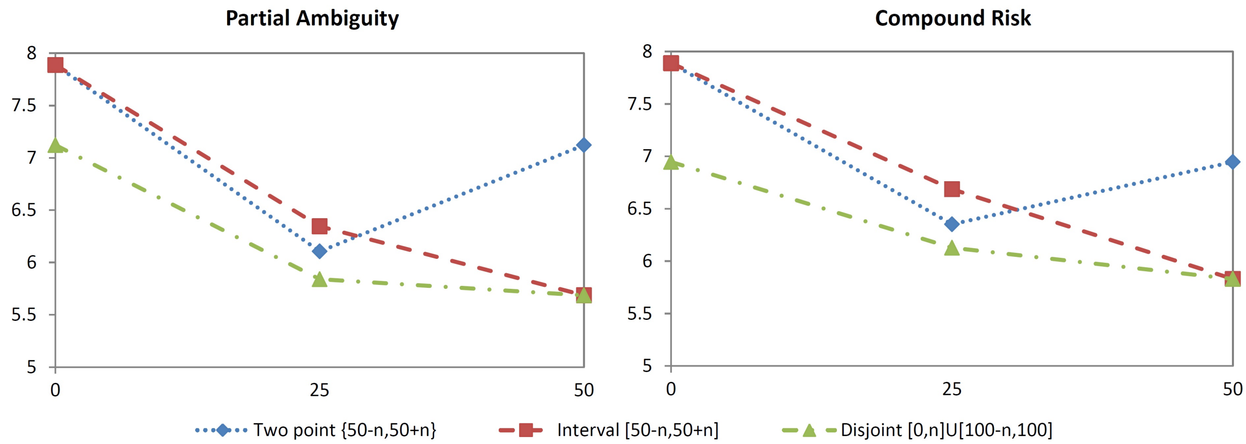



Figure 2: Mean switch points for partial ambiguity (Left) and compound risk (Right).

The resulting negative relationship is significant for both interval ambiguity ($p < 0.001$) and interval compound risk ($p < 0.001$).

**Observation 1B**: *For both disjoint ambiguity and compound risk, there is a decreasing trend in the CEs as the number of possible compositions increases.*

Using a similar analysis as is done for interval lotteries, we find that the negative relationship is significant for both disjoint ambiguity ($p < 0.001$) and disjoint compound risk ($p < 0.003$).

**Observation 1C**: *For both two-point ambiguity and compound risk, there is a decreasing trend in the CEs as spread increases except near the end-point* $\{0, 100\}$.

In the absence of a monotone pattern for two-point lotteries, we examine the pair-wise relationship among the lotteries using the Wilcoxon signed-rank test. The CE of $\{50\}$ is significantly higher than that of $\{25, 75\}$ for both ambiguity and compound lotteries ($p < 0.001$). The CE of $\{0, 100\}$ is also significantly higher than that of $\{25, 75\}$ for ambiguity ($p < 0.012$), but not for compound lotteries ($p = 0.645$). The CE of $\{50\}$ is significantly higher than that of $\{0, 100\}$ for both ambiguity and compound lotteries ($p < 0.001$).

Experiment S1 on partial ambiguity and Experiment S2 on compound risk enable us to examine whether the choice patterns observed in the main experiment could be replicated in a between-subject design (see Appendix D for details). First, Observation 1 is mostly replicated, except for two-point compound risk in Experiment S2, where we observe a decreasing trend inclusive of the end-point. Second, comparing across the three experiments, we find similar proportions of subjects exhibiting the aforementioned behavior in each of the three kinds of lotteries (see Table C.II in Appendix C). Lastly, in relation to the observed inconsistency for end-point between the main experiment and Experiment S2, we conduct a more refined comparison between $\{50\}$ with the two end-point lotteries $\{0, 100\}^A$ and $\{0, 100\}^C$. Across the three experiments and for both ambiguity and compound risk (see Table C.III in Appendix C), we find 30% to 40% of the subjects being indifferent between $\{50\}$ and $\{0, 100\}$, 30% to 40% preferring $\{50\}$ to $\{0, 100\}$, and the rest of 10% to 20% preferring $\{0, 100\}$ to $\{50\}$. This suggests that there may be two major types of subjects, namely, those satisfying time neutrality as in RRDU and those viewing the end-point as being particularly risky as in REU. A similar observation is made in Halevy (2007) when comparing $\{50\}$ and $\{0, 100\}^C$. Overall, the observed choice behaviors in the two supplementary experiments are similar to those in the main experiment.

## 4.2 Choice Patterns Across Ambiguity and Compound Risk

### 4.2.1 Correlation between Attitudes towards Ambiguity and Compound Risk

The resemblance in behavior between partial ambiguity and compound risk suggests that they may share similar underpinnings. This prompts us to examine the association between ambiguity attitude and compound risk attitude. Table I below reveals a significant association between ambiguity neutrality and RCLA (Pearson's chi-squared test, $p < 0.001$). Specifically, of 188 subjects, 31 are observed to reduce all compound lotteries, i.e., the CEs for all compound lotteries and $\{50\}$ are the same. Of these, 30 are ambiguity neural, i.e., the CEs for all ambiguity lotteries are the same as that of $\{50\}$. This is more than four times the expected frequency under the null hypothesis of independence. Among these 30 subjects, 12 switch to choosing the sure amount whenever it is at least as much as the expected value of 20, suggesting that they are risk neutral. Out of 157 subjects who do not conform to RCLA, ten are ambiguity neutral, which is about one third of the expected frequency under the null hypothesis of independence.

Table I: Association between ambiguity attitude and RCLA.

| | | Compound Risk | | |
|---|---|---|---|---|
| | | Reduction | Non-Reduction | Total |
| Ambiguity | Neutral | 30 (6.6) | 10 (33.4) | 40 |
| | Nonneutral | 1 (24.4) | 147 (123.6) | 148 |
| | Total | 31 | 157 | 188 |

Note. The two-way table presents the number of subjects by whether RCLA holds and whether ambiguity neutrality holds. Each cell indicates the number of subjects with the expected number displayed in parentheses (Pearson's chi-squared test, $p < 0.001$).

We further investigate the association between ambiguity attitude and RCLA by assessing the correlation between the ambiguity premium and the compound risk premium. The ambiguity (compound risk) premium is measured as the CE difference between the ambiguity (compound) lottery and $\{50\}$. Table C.IV in Appendix C displays the correlations between the premiums of the five ambiguity lotteries and the premiums for the corresponding compound lotteries. The average of these on-diagonal correlations is 0.588 compared with the average of 0.432 for the off-diagonal correlations between each of the five ambiguity lotteries and compound lotteries other than the corresponding one. The two correlations are significantly different ($p < 0.030$) using a test of equality of two correlation coefficients in Caci (2000). This observation further strengthens the link between ambiguity attitude and compound risk attitude for each of the five lotteries. In summary, we have the following observation regarding the correlation between ambiguity and compound lottery.

**Observation 2**: *Attitude towards partial ambiguity is significantly correlated with attitude towards the corresponding compound risk.*

#### 4.2.2 Comparison between Attitudes towards Ambiguity and Compound Risk

Observation 2 suggests the need to further explore how subjects compare subjective stage-1 priors in partial ambiguity and objective stage-1 priors in compound risk. In this subsection, we probe stage-1 priors in ambiguity lotteries by comparing the CEs for ambiguity lotteries and those of the corresponding compound lotteries. We allow for the possibility of choice error or weak preference.[17] More specifically, the CEs of two lotteries are considered equal if the absolute difference in terms of switch points does not exceed 1. With choice error 1, the number of subjects in accordance with RCLA increases to 61, and we conduct the comparison with the remaining 127 subjects not exhibiting RCLA and potentially harboring a two-stage perspective for ambiguity lotteries.

As observed earlier, under two-stage models, the CEs for ambiguity lotteries generally differ from those of the corresponding compound lotteries except for the case of subjective uniform stage-1 priors. This observation does not apply to the two-point lotteries of $\{0, 100\}$ and $\{25, 75\}$, since both REU and RRDU predict indifference across domains under the symmetry assumption. Table II displays the results of comparisons in CEs between two-point ambiguity and two-point compound risk for the 127 subjects not exhibiting RCLA. Of these subjects, 31 have similar CEs, 44 weakly prefer two-point compound risks, 43 exhibit the reverse preference, and the remaining nine subjects do not exhibit a consistent preference. The observed heterogeneity seems systematic since the comparisons for $\{0, 100\}$ and $\{25, 75\}$ are positively associated (Pearson's chi-squared test, $p < 0.024$) and the CE difference between $\{0, 100\}^A$ and $\{0, 100\}^C$ is also positively correlated with that between $\{25, 75\}^A$ and

[17]For example, the switch point at 10 suggests that the CE for lottery A is between \$19 and \$20, and the switch point at 11 suggests that the CE for lottery B is between \$20 and \$21. It is possible that \$20 is the same CE for both lotteries. In the following analysis, we classify subjects as being ambiguity neutral (performing RCLA) if the average absolute difference between each of the ambiguity lotteries (compound lotteries) and $\{50\}$ is not larger than 1.

$\{25, 75\}^C$ (spearman correlation = 0.254, p < 0.004). When making CE comparisons for the remaining three lotteries—$[25, 75]$, $[0, 25] \cup [75, 100]$ and $[0, 100]$—25 (19.7%) have similar CEs between each of the five ambiguity lotteries and the corresponding compound lotteries, 28 (22.0%) weakly prefer ambiguity lotteries to compound lotteries, 40 (31.5%) exhibit the reverse, and the remaining 34 (26.7%) do not reveal any of these patterns.

Table II: Comparison between ambiguous and compound two-point lotteries.

| | | $\{0, 100\}$ | | | |
|---|---|---|---|---|---|
| | | $CE^A < CE^C$ | $CE^A = CE^C$ | $CE^A > CE^C$ | Total |
| $\{25, 75\}$ | $CE^A < CE^C$ | 15 (9.1) | 15 (14.6) | 3 (9.4) | 33 |
| | $CE^A = CE^C$ | 14 (19.3) | 31 (30.9) | 25 (19.8) | 70 |
| | $CE^A > CE^C$ | 6 (6.6) | 10 (10.6) | 8 (6.8) | 24 |
| | Total | 35 | 56 | 36 | 127 |

Note. The two-way table presents the number of subjects in each of the categories—$CE^A < CE^C$, $CE^A = CE^C$, and $CE^A > CE^C$—indicating that the CE of a two-point ambiguity lottery may be smaller than, equal to, or larger than that of the corresponding two-point compound lottery. Each cell indicates the number of subjects with the expected number of subjects displayed in parentheses. (Pearson's chi-squared test, p < 0.024).

In summary, we have the following observation regarding the comparison between ambiguity and compound risk.

**Observation 3**: *There is heterogeneous preference between partial ambiguity and the corresponding compound risk.*

## 4.3 Individual Type Analysis

We conduct individual type analysis in terms of the proximity of the observed behavior to the predictions of specific utility models. We first differentiate subjects according to whether they conform to RCLA. With choice error 1, 61 subjects are in accordance with RCLA. Among these 61 subjects, 46 are ambiguity neutral and in accordance with SEU, and 15 are compatible with models with RCLA including multiple-prior models. For the other 127 subjects who do not exhibit RCLA, we classify them either as REU or RRDU type as follows.

Given the potential incidence of non-uniform priors or source preference in the domain of ambiguity, it is not straightforward to conduct cross-domain analysis in classifying individual choice patterns. Thus, we focus on within-domain choice patterns in arriving at individual types for REU and RRDU. In each domain, since we have three lotteries for each kind of partial ambiguity and compound risk, this classification scheme yields $3 \times 3$ pairs of possible binary comparisons. For these nine comparisons, as each model predicts specific choice patterns (as shown in subsection 3.1), we count the number of consistent choice patterns of each subject for each set of theoretical predictions, and associate the subject with the type delivering the highest number of consistent choices.

Under partial ambiguity, 58 subjects are of RRDU type with 52 having convex probability weighting functions and six having the reverse, 48 subjects are of REU type with 36 having concave stage-1 relative curvature function and 12 having the reverse, and 21 subjects are unclassified. Under compound risk, 54 subjects are of RRDU type with 45 having convex probability weighting functions and nine having the reverse, 53 subjects are of REU type with 38 having concave stage-1 relative curvature function and 15 having the reverse, and 20 subjects are unclassified.[18] Similar proportions are observed in Halevy (2007), in which the classification of individual types is conducted by combining ambiguity and compound lotteries.

Table III: Individual types with two-stage perspective.

| | | Compound Risk | | | |
|---|---|---|---|---|---|
| | | REU | RRDU | Unclassified | Total |
| Ambiguity | REU | 27 (20.0) | 13 (20.4) | 8 (7.6) | 48 |
| | RRDU | 17 (24.2) | 32 (24.7) | 9 (9.1) | 58 |
| | Unclassified | 9 (8.8) | 9 (8.9) | 3 (3.3) | 21 |
| | Total | 53 | 54 | 20 | 127 |

Note. The two-way table presents the number of subjects classified as REU or RRDU separately for ambiguity and compound risk. Each cell indicates the number of subjects with the expected number displayed in parentheses (Pearson's chi-squared test, $p < 0.050$).

Table III reports the classification results across the domains of ambiguity and compound risk, and reveals that the types are correlated (Pearson's chi-squared test, $p < 0.050$; Fisher's exact test, $p < 0.044$). For the 48 subjects classified as REU in the domain of ambiguity, 27 remain classified as REU in the domain of compound risk. For the 58 subjects classified as RRDU under ambiguity, 32 of them continue to be classified as RRDU under compound risk. These results lend further support to the two-stage perspective applied to partial ambiguity. Finally, based on the results of tests on order effects in Appendix C, we find that the observed choice patterns are robust to varying the order of presentation of ambiguity tasks and compound risk tasks.

# 5 Discussion

We begin by relating the theoretical predictions of the utility models under the three perspectives to observed choice behavior discussed in the preceding section. This is followed by further considerations including the influence of the number of the possible compositions

[18] While the behavior of some unclassified subjects could be accounted for by both REU and RRDU, the rest do not fit with either model. Alternatively, we can calculate how much individual CEs would need to change in order to be in line with the prediction of a specific model. Comparing these threshold changes to fit different predictions enables us to classify subjects into particular types and deliver similar results.

and the extent of their spread on the degree of ambiguity, robustness of the link between attitude towards ambiguity and attitude towards compound risk, and extension of the domain of partial ambiguity to include skewed ambiguity.

## 5.1 Theoretical Implications

Our main observations in the preceding section inform the models of ambiguity discussed in Section 3. The within-domain choice behavior in Observation 1 reveals a richer range of behavior than may be associated with the pessimism intuition underpinning the multiple-prior perspective on how people respond to ambiguity. Intriguingly, the observed choice behavior with respect to compound risk generally parallels what is observed for partial ambiguity, lending support to the two-stage perspective. This view is reinforced by the observed association between attitudes towards partial ambiguity and the corresponding compound risk in Observation 2. The observed heterogeneity in preference between partial ambiguity and the corresponding compound risk in Observation 3 points to the potential influence of non-uniform priors and the incidence of source preference in differentiating between subjective and objective stage-1 priors. We summarize in in Table IV the interrelations between our observed choice behavior and predictions of models under the three perspectives.

*Multiple-prior perspective.* Models under this perspective adopting the Anscombe-Aumann framework which requires RCLA for compound risks, including CEU in Schmeidler (1989) and MEU in Gilboa and Schmeidler (1989), predict that the compound lotteries in our experiment would all be indifferent to $\{50\}$. Consequently, such models are not compatible with the choice patterns in Observation 1 in the domain of compound risk, the observed association in attitudes across the two domains in Observation 2, and the heterogeneous preference between two-point ambiguity and compound risk in Observation 3. At the same time they enjoy some flexibility in accommodating the choice behavior in Observation 1 in the domain of partial ambiguity. In particular, CEU with a non-additive capacity $\nu$ possesses the most flexibility by allowing $\nu$ to be tailored to observed choice patterns, e.g., it can depend on the number of possible compositions in interval and disjoint ambiguity. It is noteworthy that models adopting a standard Savagian framework in which compound risk is absent, including Gilboa (1987) and Casadesus-Masanell, Klibanoff and Ozdenoren (2000), are silent on a possible link between attitude towards partial ambiguity and attitude towards compound risk.

*Two-stage perspective.* For two-stage models, RRDU with a convex probability weighting function is compatible with the observed aversion to increasing the number of possible compositions in both interval and disjoint ambiguity as in observations 1A and 1B as long as stage-1 priors satisfy a simple spread condition discussed in subsection 3.1. In contrast, under the same conditions, REU with a concave stage-1 relative curvature function fails to accommodate observations 1A and 1B concurrently. Instead, it implies a switch in attitude from aversion to increasing the number of possible compositions under interval ambiguity to affinity towards increasing the number of possible compositions under disjoint ambiguity. In the domain of compound risk, the overall choice patterns resemble those for partial ambiguity. Interestingly, a consistent attitude towards increasing the number of possible compositions in the domain of partial ambiguity corresponds to opposing attitudes towards stage-1 simple spread in compound risk. The objectively implemented stage-1 priors enable us to examine some axioms underpinning the two-stage models. In exhibiting stage-1 betweenness, REU implies that $[0, 100]^C$ is intermediate in preference between an interval compound lottery and its complementary disjoint compound lottery, which runs counter to $[0, 100]^C$ being the least valued lottery. For two-point compound lotteries, preferring $\{50\}$ over $\{0, 100\}^C$ in Observation 1C violates time neutrality, which is a characteristic property of RRDU.[19] Comparing across domains in Observation 2, the observed association between ambiguity attitude and compound risk attitude is compatible with adopting a two-stage perspective in modeling ambiguity aversion coupled with the failure of RCLA. This is consistent with the results of individual type analysis, which reveals that REU and RRDU constitute the bulk of our subjects.[20]

*Source Perspective.* Models of source preference incorporating additionally a two-stage perspective are exemplified by Nau's (2006) axiomatization of REU and Ergin and Gul's axiomatization of SPS which subsumes REU as well as RRDU as special cases. Besides exhibiting similar predictions as RRDU, the SPS representation can accommodate distinct non-expected utility preferences across stages, e.g., RRDU with different probability weighting functions for the two stages. This enables SPS to account for the observed time non-neutrality in Observation 1C. In Observation 3, the observed heterogeneity in preference between partial ambiguity and the corresponding compound risk suggests that subjects may have non-uniform priors or source preference between subjective stage-1 prior in partial ambiguity and objective stage-1 prior in compound risk. It is noteworthy that both stage-1 priors for two-point ambiguity and two-point compound risk are necessarily even-chance under symmetry. Should

[19]The evidence on time neutrality is mixed. A similar preference for $\{50\}$ over $\{0, 100\}^C$ is reported in Aydogan, Bleichrodt, and Gao (2016) while the reverse is observed in Budescu and Fischer (2001).

[20]We note that the observed association does not imply causality. As discussed earlier, one possibility is that a decision maker may view an ambiguous lottery as a compound lottery through a subjective stage-1 prior and exhibit ambiguity non-neutrality due to the failure of RCLA. Another possibility is for subjects to see compound lotteries as being ambiguous due to factors such as complexity. Taking the latter scenario to the extreme, the compound lotteries whose objective stage-1 priors have the same support would map to the same partial ambiguity lottery. This would in turn give rise to the same CEs regardless of the initial distribution of priors for the compound lotteries.

symmetry hold, the observed non-indifference between two-point ambiguity and two-point compound risk points to the incidence of source preference among stage-1 priors. This behavior cannot be accounted for by the existing models of ambiguity preference and points to the need for further theoretical development.

Table IV: Relating observations to models under the three perspectives.

| Observation | SEU | CEU | REU | RRDU | SPS |
|---|---|---|---|---|---|
| 1A & 1B | – | ✓ | ✓ | ✓ | ✓ |
| 1C | – | ✓ | – | – | ✓ |
| 2 | – | – | ✓ | ✓ | ✓ |
| 3 | – | – | ✓ | ✓ | ✓ |

Note. This table summarizes the implications of the main observations on the models discussed. A darker "✓" refers a model being fully compatible with an observation. A lighter "✓" refers a model being compatible with parts of an observation. A "–" indicates incompatibility between a model and an observation. CEU is compatible with Observation 1 in adapting the $\nu$ function to the observed choice pattern. REU is compatible with Observation 1A and 1B separately but not jointly. REU, RRDU, and SPS are compatible with Observation 3 incorporating a source preference at stage-1.

## 5.2 Further Considerations

*Degree of ambiguity.* With regard to Ellsberg's (1961) view of ambiguity as "a quality depending on the amount, type, reliability and unanimity of information", Becker and Brownson (1964) propose a quantitative measure of the degree of ambiguity using the range of the possible numbers of red or black cards in interval ambiguity. Two alternative measures arise naturally in our experimental setting: the number of possible compositions, and the extent of spread which can be assessed by the variance of the set of possible numbers of red or black cards. It is clear that the measure of range by itself cannot concurrently account for the influence of both the measures of number and extent of spread. In a linear regression, we investigate how the two factors of number and spread jointly affect subjects' attitude towards the three forms of partial ambiguity and find that subjects are averse to increases in both number ($p < 0.001$) and spread ($p < 0.046$).

*Robustness of link between ambiguity and compound risk.* The correlation between ambiguity attitude and RCLA has been recently investigated in Dean and Ortoleva (2012), Abdellaoui, Klibanoff and Placido (2015) and Gillen, Snowberg and Yariv (2015) following Halevy (2007). For an overall assessment, we examine the correlations for these recent studies, together with the current study and unpublished data in a study with 3146 subjects (see Table C.V in Appendix C). Compared to the threshold of 0.61 for substantial correlation as suggested in Landis and Koch (1977), the correlation coefficients in these studies range from 0.440 to 0.850 (mean: 0.598; median: 0.580). The overall significant correlation across studies reinforces the similarity in choice behavior towards partial ambiguity and compound risk.

*Factors underlying heterogeneous preference between ambiguity and compound risk.* In addition to source preference, other factors may contribute to the observed heterogeneity in preference across the domains of partial ambiguity and compound risk. One factor concerns whether stage-1 priors in ambiguity lotteries are uniform. In this regard, stage-1 priors for ambiguity lotteries may more likely be unimodal than uniform, given the nominally higher correlation of 0.557 between full ambiguity premium and compound risk premium with hypergeometric prior than the corresponding correlation of 0.483 with uniform prior as computed from data provided in Abdellaoui, Klibanoff, and Placido (2015) (see Table C.V in Appendix C). Another factor has to do with the symmetry assumption, typically adopted in studies of ambiguity. In our setting, asymmetry in stage-1 priors for ambiguity lotteries can contribute to subjects favoring partial ambiguity over the corresponding compound risk. The empirical validity of the symmetry assumption has been supported in two recent studies in Abdellaoui et al. (2011) and Epstein and Halevy (2014), and further supported by a supplementary experiment we have conducted (see Table C.VI in Appendix C).

*Skewed ambiguity and beyond.* Beyond symmetric ambiguity, the implications of ambiguity models have been examined in various other settings (Machina, 2009; Baillon, L'Haridon, and Placido, 2011; Baillon and Bleichrodt, 2015; Yang and Yao, 2016). Among these studies, a preference for ambiguity has been reported for skewed ambiguity (Abdellaoui et al., 2011; Baillon and Bleichrodt, 2015). Relatedly, in the domain of compound risk, as the winning probability declines, the degree of compound risk aversion is also found to be decreasing (Abdellaoui, Klibanoff and Placido 2015). Besides skewed ambiguity, partial knowledge may underpin uncertainty in broader domains including ambiguity involving three or more outcomes, ambiguity in the loss domain, and ambiguity arising from natural sources.

# REFERENCES


[1] Abdellaoui, M., A. Baillon, L. Placido, and P. P. Wakker (2011): "The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation," *American Economic Review* 101: 695–723.

[2] Abdellaoui, M., P. Klibanoff, and L. Placido (2015): "Experiments on Compound Risk in Relation to Simple Risk and to Ambiguity," *Management Science* 61: 1306–1322.

[3] Amarante, M., Y. Halevy, and E. Ozdenoren (2011): "Two-Stage Probabilistic Sophistication," *working paper.*

[4] Alon, S., and D. Schmeidler (2014): "Purely Subjective Maxmin Expected Utility," *Journal of Economic Theory*, 152: 382–412.

[5] Anscombe, F. J., and R. J. Aumann (1963): "A Definition of Subjective Probability," *Annals of mathematical statistics* 34: 199–205.

[6] Aydogan, I., Bleichrodt, H., and Y. Gao (2016): "An experimental test of reduction invariance," *Journal of Mathematical Psychology, forthcoming.*

[7] Baillon, A., and H. Bleichrodt (2015): "Testing ambiguity models Through the Measurement of Probabilities for Gains and Losses," *American Economic Journal: Microeconomics*, 7: 77–100.

[8] Baillon, A., O. L'Haridon, and L. Placido (2011): "Ambiguity Models and the Machina Paradoxes," *American Economic Review* 101: 1547–1560.

[9] Baillon, A., Y. Halevy, and C. Li (2014): "Experimental Elicitation of Ambiguity Attitude using the Random Incentive System," *working paper*.

[10] Becker, S. W., and F. O. Brownson (1964): "What Price Ambiguity? Or the Role of Ambiguity in Decision-Making," *Journal of Political Economy* 72: 62–73.

[11] Becker, G. M., M. H. DeGroot, and J. Marschak (1964): "Measuring Utility by a Single Response Sequential Method," *Behavioral Science* 9: 226–232.

[12] Budescu, D. V., and I. Fischer (2001): "The same but different: an empirical investigation of the reducibility principle," *Journal of Behavioral Decision Making* 14: 187C206

[13] Caci, H. M. (2000): CORTESTI: Stata Module to Test Equality of Two Correlation Coefficients. *Statistical Software Components*.

[14] Casadesus-Masanell, R., P. Klibanoff, and E. Ozdenoren (2000): "Maxmin Expected Utility over Savage Acts with a set of Priors," *Journal of Economic Theory*, 92: 35–65.

[15] Chew, S. H. (1983): "A Generalization of the Quasilinear Mean with Applications to the Measurement of Income Inequality and Decision Theory Resolving the Allais Paradox," *Econometrica* 51: 1065–1092.

[16] Chew, S.H. (1989): "Axiomatic Utility Theories With the Betweenness Property," *Annals of Operations Research* 19: 273–298.

[17] Chew, S. H., Epstein, L. G., and U. Segal (1991): "Mixture Symmetry and Quadratic Utility," *Econometrica* 59: 139–163.

[18] Chew, S. H., and J. S. Sagi (2008): "Small Worlds: Modeling Preference over Sources of Uncertainty," *Journal of Economic Theory* 139: 1–24.

[19] Dean, M., and P. Ortoleva (2012): "Allais, Ellsberg, and Preferences for Hedging," *working paper*.

[20] Dekel, E. (1986): "An Axiomatic Characterization of Preferences under Uncertainty: Weakening the Independence Axiom," *Journal of Economic Theory* 40: 304–318.

[21] Ellsberg, D. (1961): "Risk, Ambiguity, and the Savage Axioms," *Quarterly Journal of Economics* 75: 643–669.

[22] Epstein, L. G. (2010): "A Paradox for the 'Smooth Ambiguity' model of Preference," *Econometrica* 78: 2085–2099.

[23] Epstein, L. G., and Y. Halevy (2014): "No Two Experiments Are Identical," *working paper*.

[24] Ergin, H., and F. Gul (2009): "A Theory of Subjective Compound Lotteries," *Journal of Economic Theory* 144: 899–929.

[25] Fox, C. R., and A. Tversky (1995): "Ambiguity Aversion and Comparative Ignorance," *The Quarterly Journal of Economics* 110: 585–603.

[26] Fox, C. R., and A. Tversky (1998): "A Belief-Based Account of Decision under Uncertainty," *Management Science* 44: 879–895.

[27] Gillen, B., E. Snowberg., and L. Yariv (2015): "Experimenting with Measurement Error: Techniques with Applications to the Caltech Cohort Study," *working paper*.

[28] Gilboa, I. (1987): "Expected Utility with Purely Subjective Nonadditive Probabilities," *Journal of Mathematical Economics* 16: 65–88.

[29] Gilboa, I., and D. Schmeidler (1989): "Maxmin expected Utility with Non-unique Prior," *Journal of Mathematical Economics* 18: 141–153.

[30] Halevy, Y. (2007): "Ellsberg Revisited: An Experimental Study," *Econometrica* 75: 503–536.

[31] Halevy, Y., and V. Feltkamp (2005): "A Bayesian Approach to Uncertainty Aversion," *The Review of Economic Studies*, 72: 449–466.

[32] Keynes, J. M. (1921): *A treatise on probability.* London: MacMillan.

[33] Klibanoff, P., M. Marinacci, and S. Mukerji (2005): "A Smooth Model of Decision Making under Ambiguity," *Econometrica* 73: 1849–1892.

[34] Klibanoff, P., M. Marinacci, and S. Mukerji (2012): "On the Smooth Ambiguity Model: A reply," *Econometrica* 80: 1303–1321.

[35] Knight, F. (1921): *Risk, Uncertainty and Profit.* Houghton Mifflin, Boston, New York.

[36] Kreps, D. M., and E. L. Porteus (1978): "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46: 185–200.

[37] Landis, J. R., and G. G. Koch (1977): "The Measurement of Observer Agreement for Categorical Data," *Biometrics* 33: 159–174.

[38] Machina, M. J. (1982): "Expected Utility Analysis without the Independence Axiom," *Econometrica*: 50: 277–323.

[39] Machina, M. J. (2009): "Risk, Ambiguity, and the Rank-Dependence Axioms," *American Economic Review* 99: 385–92.

[40] Machina, M. J., and D. Schmeidler (1992): "A more robust definition of subjective probability," *Econometrica* 60: 745–780.

[41] Nau, R. F. (2006): "Uncertainty Aversion with Second-Order Utilities and Probabilities," *Management Science* 52: 136–145.

[42] Quiggin, J. (1982): "A Theory of Anticipated Utility," *Journal of Economic Behavior and Organization* 3: 323–343.

[43] Schmeidler, D. (1989): "Subjective Probability and Expected Utility without Additivity," *Econometrica* 57: 571–587.

[44] Segal, U. (1987): "The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach," *International Economic Review* 28: 175–202.

[45] —— (1990): "Two-Stage Lotteries without the Reduction Axiom," *Econometrica* 58: 349–377.

[46] Seo, K. (2009): "Ambiguity and Second-Order Belief," *Econometrica* 77: 1575–1605.

[47] Wakker, P. P. (1987): "Nonadditive Probabilities and Derived Strengths of Preferences," *unpublished manuscript.*

[48] Yang, C. L., and L. Yao (2016): "Testing Ambiguity Theories in a New Experimental Design with Mean-preserving Prospects," *Quantitative Economics, forthcoming.*

# Supplement to "Partial Ambiguity"

## Appendix A: Further Theoretical Analysis

We first analyze the theoretical prediction for Maxmin expected utility and follow up models under multiple prior perspective in the domain of partial ambiguity. Next, we derive the detailed analysis for recursive rank dependent utility for the three forms of partial ambiguity. Lastly, we analyze the theoretical prediction for Chew and Sagi's (2008) source preference model.

### A.1 Maxmin expected utility and follow up models

*Maxmin expected utility.* The maxmin expected utility (MEU) in Gilboa and Schmeidler (1989) evaluates an ambiguous lottery $L_i^A$ with the expected utility corresponding to the worst prior in a convex set of priors $\Pi_{L_i^A}$ as follows:

$$U_{MEU}(L_i^A) = \min_{\mu \in \Pi_{L_i^A}} \mu\left(R_{L_i^A}\right) u\left(w\right).$$

As indifference between betting on red and black implies that $\Pi_{L_i^A}$ is symmetric, MEU exhibits global ambiguity aversion: $\{50\}$ is preferred to any ambiguous lottery $L_i$. It should be noted that the behavior of the set of priors $\Pi_{L_i}$ is inherently flexible, and MEU can account for a wide range of ambiguity averse choice behavior with a judicious choice of the worst prior in each $\Pi_{L_i^A}$.

*Variational Preference.* Maccheroni, Marinacci and Rustichini (2006) propose an alternative and more flexible generalization of MEU as follows:

$$U_{VP}\left(L_i^A\right) = \min_{\mu \in \Delta} \left\{\mu\left(R_{L_i^A}\right) u\left(w\right) + a_{L_i^A}\left(\mu\right)\right\},$$

where $\Delta$ refers to the set of all possible priors and $a_{L_i}\left(\mu\right) : \Delta \to [0, \infty)$ is an index of ambiguity aversion. Notice that VP reduces to MEU if $a_{L_i^A}$ is an indicator function for $\Pi_{L_i^A}$, and it follows that VP inherits the predictions of MEU in the domain of partial ambiguity. The same qualitative behavior also applies to the contraction model (Gajdos et al., 2008), which delivers a weighted combination between SEU and MEU with built-in ambiguity aversion.

*$\alpha$-maxmin expected utility.* Ghirardato, Maccheroni, and Marinacci (2004) axiomatize $\alpha$-maxmin expected utility ($\alpha$-MEU) that delivers a linear combination of maxmin EU and maxmax EU as follows:

$$U_{\alpha-MEU}\left(L_i^A\right) = \alpha \min_{\mu \in \Pi_{L_i^A}} \mu\left(R_{L_i^A}\right) u\left(w\right) + (1-\alpha) \max_{\mu \in \Pi_{L_i^A}} \mu\left(R_{L_i^A}\right) u\left(w\right).$$

Depending on the value of $\alpha$, this model is highly flexible and can selectively exhibit ambiguity tolerance. The same predictions apply to Siniscalchi's (2009) vector expected utility model which incorporates an adjustment function in addition to SEU.

### A.2 Recursive rank dependent utility

*Interval Ambiguity $\mathbf{I}_n^A$.* The utility for an interval ambiguity lottery $\mathbf{I}_n^A$ under uniform prior is given by:

$$U(\mathbf{I}_n^A) = \sum_{i=50-n}^{50+n} f\left(1 - \frac{i}{100}\right) \left[f\left(\frac{i+1-(50-n)}{2n+1}\right) - f\left(\frac{i-(50-n)}{2n+1}\right)\right].$$

This can be approximated using a uniform random variable over $[0,1]$ with cumulative distribution function $F$ as follows:

$$U(\mathbf{I}_n^A) = \int_{0.5-\widehat{n}}^{0.5+\widehat{n}} f(s)\, d\left(-f\left(1-F(s)\right)\right),$$

where $\widehat{n}$ takes the values between 0 and 0.5, and $F = \frac{s+\widehat{n}-0.5}{2\widehat{n}}$ for $s \in [0.5-\widehat{n}, 0.5+\widehat{n}]$. Let $x = \frac{s+\widehat{n}-0.5}{2\widehat{n}}$. We have:

$$U = \int_0^1 -f\left(2\widehat{n}x + 0.5 - \widehat{n}\right) df\left(1-x\right).$$

Differentiating with respect to $\widehat{n}$ yields:

$$U' = \int_0^1 (2x-1)\, f'\left((1-2\widehat{n})\, x + \widehat{n}\right) f'\left(1-x\right) dx.$$

Evaluating $U'$ at $\widehat{n}=0$ gives:

$$U'|_{\widehat{n}=0} = \int_0^1 (2x-1)\, f'(x)\, f'(1-x)\, dx,$$

which can be rewritten as:

$$-\int_0^{0.5} (1-2x)\, f'(x)\, f'(1-x)\, dx + \int_{0.5}^1 (2x-1)\, f'(x)\, f'(1-x)\, dx, \tag{A.1}$$

which equals 0 given the symmetry of the two terms after changing the variable in the second term to $1-x$.

Observe that $f\left(2\widehat{n}x + 0.5 - \widehat{n}\right) > f(x)$ when $x < 0.5$ since $2\widehat{n}x + 0.5 - \widehat{n} > x$. Similarly, we have $f\left(2\widehat{n}x + 0.5 - \widehat{n}\right) < f(x)$ when $x > 0.5$. It follows that when $f$ is convex, $U' < 0$, i.e., aversion to increasing the number of possible compositions in interval ambiguity, since changing $f'(x)$ to $f'\left(2\widehat{n}x + 0.5 - \widehat{n}\right)$ will increase the first term of (A.1) and decrease its second term.■

*Disjoint Ambiguity* $\mathbf{D}_n^A$. The utility $U(\mathbf{D}_n^A)$ for a disjoint ambiguity lottery $\mathbf{D}_n^A$ under uniform prior is given by:

$$\sum_{i=0}^{n} f\left(1-\tfrac{i}{100}\right)\left[f\left(\tfrac{i+1}{2(n+1)}\right) - f\left(\tfrac{i}{2(n+1)}\right)\right] + \sum_{i=n+1}^{2n+1} f\left(\tfrac{2n+1-i}{100}\right)\left[f\left(\tfrac{i+1}{2(n+1)}\right) - f\left(\tfrac{i}{2(n+1)}\right)\right].$$

This can be approximated using a uniform random variable over $[0,1]$ with cumulative distribution function $F$ as follows:

$$U = \int_0^{\widehat{n}} f(s)\, d\left(-f\left(1-F(s)\right)\right) + \int_{1-\widehat{n}}^1 f(s)\, d\left(-f\left(1-F(s)\right)\right),$$

where $\widehat{n}$ takes the values from 0.5 to 0 in disjoint ambiguity, and $F(s) = \frac{s}{2\widehat{n}}$ for $s \in [0, \widehat{n}]$ and $F(s) = \frac{s-(1-2\widehat{n})}{2\widehat{n}}$ for $s \in [1-\widehat{n}, 1]$. Letting $x$ equal $\frac{s}{2\widehat{n}}$ in the first integral and equal $\frac{s-(1-2\widehat{n})}{2\widehat{n}}$ in the second integral. We have:

$$U = \int_0^{0.5} -f\left(2\widehat{n}x\right) df\left(1-x\right) - \int_{0.5}^1 f\left(2\widehat{n}x + (1-2\widehat{n})\right) df\left(1-x\right).$$

Differentiating with respect to $\widehat{n}$ yields:

$$U' = \int_0^{0.5} 2x f'(2\widehat{n}x) f'(1-x)\, dx + \int_{0.5}^{1} (2x-2) f'(2\widehat{n}x + (1-2\widehat{n})) f'(1-x)\, dx.$$

Evaluating $U'$ at $\widehat{n} = 0.5$ gives:

$$U'|_{\widehat{n}=0} = \int_0^{0.5} 2x f'(x) f'(1-x)\, dx + \int_{0.5}^{1} (2x-2) f'(x) f'(1-x)\, dx, \tag{A.2}$$

which again equals 0 given the symmetry of the two terms.

Observe that $f(2\widehat{n}x) < f(x)$ when $x < 0.5$ and $f(2\widehat{n}x + (1-2\widehat{n})) > f(x)$ when $x > 0.5$. It follows that when $f$ is convex, $U' < 0$, i.e., aversion to increasing the number of possible compositions in disjoint ambiguity, since changing $f'(x)$ to $f'(2\widehat{n}x)$ will decrease the first term of (B.2) while changing $f'(x)$ to $f'(2\widehat{n}x + (1-2\widehat{n}))$ will increase the second term of (A.2).■

*Two-point Ambiguity* $\mathbf{T}_n^A$. The utility for a two-point ambiguity lottery $\mathbf{T}_n^A$ under uniform prior is given by:

$$U_{RRDU}(\mathbf{T}_n) = (1 - f(0.5)) f(0.5 - \widehat{n}) + f(0.5) f(0.5 + \widehat{n}),$$

Differentiating with respect to $\widehat{n}$ yields:

$$U' = f(0.5) f'(0.5 + \widehat{n}) - (1 - f(0.5)) f'(0.5 - \widehat{n}).$$

Given $f$ convex, we have $U'|_{\widehat{n}=0} < 0$ and $U'|_{\widehat{n}=0.5} > 0$, which in turn implies $U' < 0$ for $\widehat{n}$ small, and $U' > 0$ as $\widehat{n}$ approaches 0.5.■

*Stage-1 Spread*. Given two ambiguous lotteries represented by stage-1 priors with cumulative distribution functions $F$ and $G$ such that $G$ is a stage-1 spread of $F$, consider the difference:

$$\int f(x)\, d(-f(1 - F(x))) - \int f(x)\, d(-f(1 - G(x))).$$

This becomes $\int f'(x) [f(1 - F(x)) - f(1 - G(x))]\, dx$ after integrating by parts. We have that $f(1 - F(x)) - f(1 - G(x)) \geq 0$ for $x < 0.5$ and $f(1 - F(x)) - f(1 - G(x)) \leq 0$ for $x > 0.5$, since $F(x) \leq G(x)$ for $x < 0.5$ and $F(x) \geq G(x)$ for $x > 0.5$. Changing variables yields:

$$\int_0^{0.5} f'(x) [f(1 - F(x)) - f(1 - G(x))]\, dx - \int_0^{0.5} f'(1-x) [f(1 - G(1-x)) - f(1 - F(1-x))]\, dx.$$

We proceed to compare $\frac{f'(x)}{f'(1-x)}$ and $\frac{f(1-G(1-x)) - f(1-F(1-x))}{f(1-F(x)) - f(1-G(x))}$. Since $f$ is convex, we have

$$f'(1 - F(1-x)) \leq \frac{f(1 - G(1-x)) - f(1 - F(1-x))}{F(1-x) - G(1-x)} \leq f'(1 - G(1-x)),$$

and

$$f'(1 - F(x)) \geq \frac{f(1 - F(x)) - f(1 - G(x))}{G(x) - F(x)} \geq f'(1 - G(x)).$$

Given symmetry of $F$ and $G$, $F(1-x) - G(1-x) = G(x) - F(x)$. Dividing the above two inequalities yields

$$\frac{f'(1 - F(1-x))}{f'(1 - F(x))} \leq \frac{f(1 - G(1-x)) - f(1 - F(1-x))}{f(1 - F(x)) - f(1 - G(x))} \leq \frac{f'(1 - G(1-x))}{f'(1 - G(x))}.$$

Suppose $F$ is a stage-1 spread of uniform prior, then $\frac{f'(1-F(1-x))}{f'(1-F(x))} \geq \frac{f'(x)}{f'(1-x)}$ for $x \leq 0.5$. It follows that the decision maker prefers $G$ to $F$ if both of them are spreads of uniform prior. Conversely, suppose uniform prior is a stage-1 spread of $G$, then $\frac{f'(1-G(1-x))}{f'(1-G(x))} \leq \frac{f'(x)}{f'(1-x)}$ at $x \leq 0.5$ and we have the decision maker preferring $F$ to $G$ if uniform prior is a stage-1 spread of both $F$ and $G$.■

### A.3 Source preference of Chew and Sagi (2008)

Chew and Sagi's (2008) model directly distinguishes among the even-chance bets from the three primitive sources of uncertainty in our experimental design: pure risk derived from the known composition of half red and half black in $\{50\}$, full ambiguity based on the unknown compositions of red and black in $[0,100]^A$, and additionally the all-red or all-black ambiguity in $\{0,100\}^A$. These three sources generate 50-50 probabilities that may be differentiated in terms of preference. In the following, we demonstrate how the source preference approach with built-in RCLA endogenously generates a two-stage representation for the various forms of partial ambiguity.

*Interval Ambiguity* $\mathbf{I}_n^A$. It comprises $100-2n$ cards with known composition—half red and half black—while the composition of the rest $2n$ cards is fully unknown. With RCLA for known probabilities, an interval ambiguous lottery induces a lottery on the overall "known" domain that delivers $(50-n)/100$ chance of getting $w$, $(50-n)/100$ chance of getting 0 and $2n/100$ chance of getting a fully ambiguous lottery, and it is represented as follows:

$$\left(\frac{50-n}{100}, w; \frac{2n}{100}, c_{[0,100]^A}; \frac{50-n}{100}, 0\right),$$

where $c_{[0,100]^A}$ is the CE for a bet on the fully unknown deck $[0,100]^A$.

*Disjoint Ambiguity* $\mathbf{D}_n^A$. It comprises $100-n$ cards that are either all red or all black, and $n$ cards with fully unknown composition. Similarly, the induced lottery on the "known" domain is:

$$\left(\frac{100-n}{100}, c_{\{0,100\}^A}; \frac{n}{100}, c_{[0,100]^A}\right),$$

where $c_{\{0,100\}^A}$ is the CE for a bet on the either-all-red-or-all-black deck $\{0,100\}^A$. Notice that the expression above converges to $(0.5, c_{\{0,100\}^A}; 0.5, c_{[0,100]^A})$ rather than $c_{[0,100]^A}$ as $n$ approaches 50. This behavior is related to an alternative description of full ambiguity $[0,100]^A$. Besides its intended interpretation of being fully unknown, it can first be described as comprising 50 cards which are either all red or all black while the composition of the other 50 cards remains unknown. This process can be applied to the latter 50 cards to arrive at a further division into 25 cards which are either all red or all black while the composition of the remaining 25 cards remains unknown. Doing this ad infinitum gives rise to a dyadic decomposition of $[0,100]^A$ into subintervals which are individually either all-red or all-black. For the source model to deliver the same CE for $[0,100]^A$, we need to restrict its evaluation to undecomposed intervals of ambiguity, and the overall expression exhibits discontinuous behavior at $n=50$.

*Two-point Ambiguity* $\mathbf{T}_n^A$. It comprises $100-2n$ cards with known composition—half red and half black—while the composition of the rest of the $2n$ cards is either all red or all black, and the induced lottery on the "known" domain is given by:

$$\left(\frac{50-n}{100}, w; \frac{2n}{100}, c_{\{0,100\}^A}; \frac{50-n}{100}, 0\right).$$

Consider the baseline SEU model with a utility index $u$ for objective risk on the "known" domain. The utility for an interval ambiguous lottery is then given by

$$\frac{50-n}{100}u\left(w\right)+\frac{2n}{100}u\left(c_{[0,100]^A}\right)+\frac{50-n}{100}u\left(0\right),$$

which is monotonically decreasing in $n$ if we have $\frac{u(w)+u(0)}{2} > u(c_{[0,100]^A})$. This latter condition corresponds to standard ambiguity aversion $\{50\} \succ [0,100]^A$ . Behaviorally, a subject exhibits aversion to increasing the number of possible compositions under the same condition.

Similarly, the SEU for a two-point ambiguous lottery is

$$\frac{50-n}{100}u\left(w\right)+\frac{2n}{100}u\left(c_{\{0,100\}^A}\right)+\frac{50-n}{100}u\left(0\right),$$

and we have aversion to two-point spread, i.e., the utility is monotonically decreasing in $n$, if $\frac{u(w)+u(0)}{2} > u(c_{\{0,100\}^A})$. This condition corresponds to a preference for betting on the known deck $\{50\}$ to betting on the deck $\{0,100\}^A$ that comprises of either all red or all black cards.

For disjoint ambiguity, we have aversion to increasing the number of possible compositions if $c_{\{0,100\}^A} > c_{[0,100]^A}$, which corresponds to a preference of $\{0,100\}^A \succ [0,100]^A$ . Note that this implication does not depend on the assumption of SEU but only on stochastic dominance. In contrast, it is possible for general non-expected utility preferences, e.g., quadratic preference (Chew, Epstein and Segal, 1991), to exhibit non-monotone behavior in interval and two-point ambiguity. See Chew, Miao and Zhong (2013) for a discussion of Chew and Sagi's (2008) source preference model without reduction.

In line with Chew and Sagi's (2008) source preference approach, Abdellaoui et al. (2011) propose a model with a source-dependent probability weighting function in conjunction with a cumulative prospect theory specification. As in the preceding exposition of source-dependent SEU, being silent on how compound lotteries may be evaluated in the absence of RCLA, this model exhibits considerable flexibility in modeling choice behavior in our setting and can distinguish among the three main sources of uncertainty should we take the view that each partial ambiguity lottery itself represents a possibly distinct source.

## Appendix B: Axioms of Compound Risk and Their Implications

Let $X_j = (p_j, x_j)_j$ denote a simple lottery paying $x_j$ with probability $p_j$, and $\mathbf{X} = \left(q^k, X^k\right)_k$ a compound lottery paying simple lottery $X^k = \left(p^k_j, x^k_j\right)_j$ with probability $q^k$. In addition, denote a degenerate lottery paying $x$ for sure by $\delta_x$. Similarly, a degenerate compound lottery paying simple lottery $X_j$ for sure is denoted by $\delta_{X_j}$ and $\left(q^j, \delta_{x_j}\right)_j$ is another kind of degenerate compound lottery which pays degenerate simple lottery $\delta_{x_j}$ with probability $q^j$. We define two operations $+$ and $\oplus$ to combine risks at two different stages. We use $+$ to denote a mixture operation for simple risks and $\oplus$ to denote a mixture operation for stage-1 risks.

Given two simple lotteries $X_m = (p_1, x_1; p_2, x_2; ...; p_m, x_m)$ and $Y_n = (q_1, y_1; q_2, y_2; ...; q_n, y_n)$ , a mixture with probability $r$, $rX_m + (1-r)Y_n$, is identified with a simple lottery given by:

$$(rp_1, x_1; ...; rp_m, x_m; (1-r)q_1, y_1; ...; (1-r)q_n, y_n).$$

In other words, $+$ can be used as a mixture operation for stage-2 risks as follows:

$$\delta_{rX_m+(1-r)Y_n} = \delta_{(rp_1,x_1;...;rp_m,x_m;(1-r)q_1,y_1;...;(1-r)q_n,y_n)}.$$

In a similar vein, given two compound lotteries $\mathbf{X} = (p^1, X^1; p^2, X^2; ...; p^m, X^m)$ and $\mathbf{Y} = (q^1, Y^1; q^2, Y^2; ...; q^n, Y^n)$, a stage-1 mixture with probability $r$, $r\mathbf{X} \oplus (1-r)\mathbf{Y}$, is identified with the compound lottery

$$\left(rp^1, X^1; ...; rp^m, X^m; (1-r)q^1, Y^1; ...; (1-r)q^1, Y^n\right).$$

To illustrate the difference between $+$ and $\oplus$, consider two compound lotteries, $\left(1, \left(\frac{1}{2}, x; \frac{1}{2}, 0\right)\right)$ and $\left(1, \left(\frac{1}{3}, y; \frac{2}{3}, 0\right)\right)$. A mixture operation $+$ with probability $\frac{1}{2}$ delivers the following:

$$\left(1, \frac{1}{2}\left(\frac{1}{2}, x; \frac{1}{2}, 0\right) + \frac{1}{2}\left(\frac{1}{3}, y; \frac{2}{3}, 0\right)\right) = \left(1, \left(\frac{1}{4}, x; \frac{1}{6}, y; \frac{7}{12}, 0\right)\right),$$

while a mixture operation $\oplus$ with probability $\frac{1}{2}$ delivers the following:

$$\frac{1}{2}\left(1, \left(\frac{1}{2}, x; \frac{1}{2}, 0\right)\right) \oplus \frac{1}{2}\left(1, \left(\frac{1}{3}, y; \frac{2}{3}, 0\right)\right) = \left(\frac{1}{2}, \left(\frac{1}{2}, x; \frac{1}{2}, 0\right); \frac{1}{2}, \left(\frac{1}{3}, y; \frac{2}{3}, 0\right)\right).$$

We now introduce several commonly used axioms in the literature on decision making involving compound risk and discuss their implications for choice behavior in our setup. We begin with the following axiom which has largely been implicit in the literature on generalizing expected utility under different weakening of its independence axiom.

*Reduction of Compound Lottery Axiom (RCLA)*: $\left(q^k, X^k\right)_k \sim q^1X^1 + q^2X^2 + ... + q^kX^k$.

RCLA requires a compound lottery $\left(q^k, X^k\right)_k$ to be indifferent to a simple lottery $q^1X^1 + q^2X^2 + ... + q^kX^k$ whose outcomes are taken from the component lotteries $X^k$ with the corresponding probabilities derived from the given compound lottery. This property is inherent to any utility model whose domain of choice comprises a set of probability measures with a convex combination of two lotteries interpreted as a compound lottery. In relaxing RCLA, people may exhibit distinct attitudes towards risks at different stages in a compound lottery. To accommodate this, we may limit the scope of a preference axiom to simple lotteries in a single-stage setting. Clearly, applying $+$ on within-stage risks does not imply that RCLA holds.

We adapt the independence axiom and the betweenness axiom to the domain of simple lotteries for both stage-1 and stage-2 risks.

*Stage-1 Independence*: $(q^m, X^m)_m \succeq (q^n, X^n)_n$ *implies that* $\alpha (q^m, X^m)_m \oplus (1-\alpha) \left(q^l, X^l\right)_l \succeq \alpha (q^n, X^n)_n \oplus (1-\alpha) \left(q^l, X^l\right)_l$.

*Stage-2 Independence*: $X_m \succeq X_n$ *implies that* $\alpha X_m + (1-\alpha) X_l \succeq \alpha X_n + (1-\alpha) X_l$.

Independence requires that the preference between two lotteries is preserved when each is mixed with a common third lottery at the same probability.

*Stage-1 Betweenness*: $(q^m, X^m)_m \succeq \alpha (q^m, X^m)_m \oplus (1-\alpha) (q^n, X^n)_n \succeq (q^n, X^n)_n$ *if* $(q^m, X^m)_m \succeq (q^n, X^n)_n$.

*Stage-2 Betweenness*: $X_m \succeq \alpha X_m + (1-\alpha) X_n \succeq X_n$ *if* $X_m \succeq X_n$.

Betweenness requires a mixture between two lotteries to be intermediate in preference between the preference for two respective lotteries.

The following axiom provides a link between stage-1 and stage-2 risk preference.

*Time Neutrality*: Given $X_j = (p_j, x_j)_j$, $\delta_{X_j} \sim \left(p^j, \delta_{x_j}\right)_j$.

This axiom requires a decision maker to be indifferent between two degenerate compound lotteries if they reduce to the same simple lottery. Put differently, whether the resolution of risks occurs at stage-1 or stage-2 does not influence the preference for degenerate compound lotteries.

It is straightforward to see that RCLA implies time neutrality and that independence implies betweenness (see Segal (1990) for a detailed discussion). For completeness in exposition, we present the following axiom which together with RCLA implies independence.

*Compound Independence Axiom*: $X^k \succeq X^{k'}$ *iff* $\left(q^1, X^1; q^2, X^2; ...; q^k, X^{k'}; ...; q^n, X^n\right) \succeq \left(q^1, X^1; q^2, X^2; ...; q^k, X^{k'}; ...; q^n, X^n\right)$.

Compound independence requires a compound lottery $\left(q^k, X^k\right)_k$ to becomes less preferred if any of the component simple lotteries is replaced with a less preferred simple lottery.

We summarize below the implications of RCLA, stage-1 betweenness and time neutrality for choice behavior in our experiments.

**Implication R:** RCLA implies that all compound lotteries are indifferent to $\{50\}$.

This follows from observing that the compound lotteries in our setup reduce to $\{50\}$.

**Implication B:** Stage-1 Betweenness implies that $[0, 100]^C$ is ranked between $[n, 100 - n]^C$ and $[0, n] \cup [100 - n, 100]^C$.

This follows from observing that $[0, 100]^C = \frac{2n}{100}[n, 100 - n]^C \oplus \frac{100-2n}{100}[0, n] \cup [100 - n, 100]^C$ in our setup, assuming the overlapping two points $\{n\}$ and $\{100 - n\}$ are negligible.

**Implication T:** Time Neutrality implies that $\{50\} \sim \{0, 100\}^C$.

This follows from observing that $\{50\}$ and $\{0, 100\}^C$ reduce to the same lottery.

# Appendix C: Supplementary Tables and Figures

## C.1 Tables

Table C.I Summary statistics in main experiment.

| | N | Mean | SD | Seeking | Neutrality | Aversion |
|---|---|---|---|---|---|---|
| $\{50\}$ | 188 | 7.888 | 4.716 | 27 | 19 | 142 |
| $[25, 75]^A$ | 188 | 6.346 | 4.314 | 30 | 65 | 93 |
| $[0, 25] \cup [75, 100]^A$ | 188 | 5.840 | 4.297 | 19 | 65 | 104 |
| $\{25, 75\}^A$ | 188 | 6.106 | 4.254 | 24 | 70 | 94 |
| $[0, 100]^A$ | 188 | 5.686 | 4.483 | 22 | 69 | 97 |
| $\{0, 100\}^A$ | 188 | 7.122 | 5.665 | 34 | 80 | 74 |
| $[25, 75]^C$ | 188 | 6.686 | 4.256 | 29 | 78 | 81 |
| $[0, 25] \cup [75, 100]^C$ | 188 | 6.128 | 4.199 | 39 | 54 | 95 |
| $\{25, 75\}^C$ | 188 | 6.351 | 4.194 | 35 | 57 | 96 |
| $[0, 100]^C$ | 188 | 5.830 | 4.259 | 25 | 63 | 100 |
| $\{0, 100\}^C$ | 188 | 6.947 | 5.866 | 37 | 72 | 79 |

Note. This table summarizes the mean and standard deviation of CEs, as well as the number of subjects who exhibit aversion, neutrality and affinity towards each lottery.

Table C.II Trends in choice patterns across experiments

| A: Interval Lotteries | | | | |
|---|---|---|---|---|
| | Ambiguity (M) | Compound Risk (M) | Ambiguity (S1) | Compound Risk (S2) |
| Increasing | 12 | 15 | 0 | 12 |
| Constant | 49 | 47 | 20 | 21 |
| Decreasing | 79 | 85 | 25 | 30 |
| **B: Disjoint Lotteries** | | | | |
| | Ambiguity (M) | Compound Risk (M) | Ambiguity (S1) | Compound Risk (S2) |
| Increasing | 33 | 42 | 4 | 22 |
| Constant | 57 | 52 | 19 | 16 |
| Decreasing | 50 | 50 | 19 | 26 |
| **C: Two-point Lotteries** | | | | |
| | Ambiguity (M) | Compound Risk (M) | Ambiguity (S1) | Compound Risk (S2) |
| Increasing | 17 | 20 | 7 | 8 |
| Constant | 54 | 41 | 22 | 14 |
| Decreasing 1 | 50 | 45 | 33 | 17 |
| Decreasing 2 | 53 | 60 | 22 | 18 |

Note. Panel A (B) displays the number of subjects whose CEs are increasing, constant or decreasing as the number of possible compositions increases for the interval (disjoint) lotteries. Panel C displays the number of subjects whose CEs are increasing, constant, or decreasing in two ways (Decreasing 1: including the end point; Decreasing 2: excluding the end point) as the spread increases for the two-point lotteries.

Table C.III End-point behavior across experiments

| Experiment | $\{50\} > \{0,100\}^A$ | $\{50\} = \{0,100\}^A$ | $\{50\} < \{0,100\}^A$ |
|---|---|---|---|
| Main | 74 (39.4%) | 80 (42.6%) | 34 (18.1%) |
| S1 | 33 (31.1%) | 49 (46.2%) | 24 (22.6%) |
| | $\{50\} > \{0,100\}^C$ | $\{50\} = \{0,100\}^C$ | $\{50\} < \{0,100\}^C$ |
| Main | 79 (42.0%) | 72 (38.3%) | 37 (19.7%) |
| S2 | 43 (43.9%) | 34 (34.7%) | 21 (21.4%) |

Note. This table displays the number (percentage) of subjects in terms of their preference between $\{50\}$ and $\{0,100\}^A$ in the main experiment and experiment S1, as well as between $\{50\}$ and $\{0,100\}^C$ in the main experiment and experiment S2.

Table C.IV Spearman correlation of CEs between ambiguity and compound risk

| | $[25,75]^C$ | $[0,25] \cup [75,100]^C$ | $\{25,75\}^C$ | $[0,100]^C$ | $\{0,100\}^C$ |
|---|---|---|---|---|---|
| $[25,75]^A$ | **0.614** | 0.525 | 0.544 | 0.492 | 0.267 |
| $[0,25] \cup [75,100]^A$ | 0.477 | **0.613** | 0.597 | 0.520 | 0.451 |
| $\{25,75\}^A$ | 0.564 | 0.556 | **0.694** | 0.600 | 0.382 |
| $[0,100]^A$ | 0.497 | 0.590 | 0.473 | **0.580** | 0.285 |
| $\{0,100\}^A$ | 0.170 | 0.271 | 0.378 | 0.199 | **0.441** |

TABLE C.V Correlation between ambiguity and compound risk premiums across studies.

| Study | Correlation | N | Remark |
|---|---|---|---|
| Halevy (2007)[a] | 0.474 | 104 | first round |
| Halevy (2007)[a] | 0.810 | 38 | robustness round |
| Dean and Ortoleva (2012) | 0.730 | 190 | - |
| Abdellaoui, Klibanoff and Placido (2015)[a] | 0.483 | 115 | uniform compound risk |
| Abdellaoui, Klibanoff and Placido (2015)[a] | 0.557 | 115 | hypergeometric compound risk |
| Gillen, Snowberg and Yariv (2015) | 0.440 | 786 | no control for measurement error |
| Gillen, Snowberg and Yariv (2015) | 0.850 | 786 | control for measurement error |
| Current study | 0.580 | 188 | $[0,100]$ |
| Current study | 0.614 | 188 | $[25,75]$ |
| Current study | 0.613 | 188 | $[0,25] \cup [75,100]$ |
| Current study | 0.694 | 188 | $\{25,75\}$ |
| Current study | 0.441 | 188 | $\{0,100\}$ |
| Unpublished data[b] | 0.489 | 3146 | $[0,100]$ |

[a]The correlations are calculated based on data provided.

[b]This is from an unpublished study of choice behavior in 2010 and 2011 conducted by Chew and Zhong together with Richard P. Ebstein.

TABLE C.VI Color preference across studies.

| Study | color preference | N | Note |
|---|---|---|---|
| Abdellaoui et al. (2011) | 1% | 67 | eight-color urn |
| Epstein and Halevy (2015) | 5% | 80 | first experiment |
| Epstein and Halevy (2015) | 23% | 87 | second experiment |
| Epstein and Halevy (2015) | 14% | 44 | risk control with $\{50\}$ |
| Epstein and Halevy (2015) | 23% | 39 | single urn control with $[0, 100]^A$ |
| Current study | 13% | 39 | $\{50\}$ |
| Current study | 15% | 39 | $[0, 100]^A$ |
| Current study | 15% | 39 | $\{0, 100\}^A$ |

Note. Abdellaoui et al. (2011) test symmetry for one-color events, two-color events, and four-color events in an eight-color unknown urn. Epstein and Halevy (2014) test symmetry with two unknown urns in two experiments, along with one risky urn and one fully ambiguous urn as controls. Our supplementary experiment follows Halevy and Epstein (2014) in which subjects make binary choices between betting on either color with slightly unequal prizes.

TABLE C.VII Association between ambiguity attitude and RCLA.

Panel A: Ambiguity first

| Ambiguity | | Compound Risk: Reduction | Compound Risk: Non-Reduction | Total |
|---|---|---|---|---|
| | Neutral | 18 (3.7) | 2 (16.3) | 20 |
| | Nonneutral | 1 (15.3) | 81 (66.7) | 82 |
| | Total | 19 | 83 | 102 |

Panel B: Compound risk first

| Ambiguity | | Compound Risk: Reduction | Compound Risk: Non-Reduction | Total |
|---|---|---|---|---|
| | Neutral | 12 (2.8) | 8 (17.2) | 20 |
| | Nonneutral | 0 (9.2) | 66 (56.8) | 66 |
| | Total | 12 | 74 | 86 |

Note. The two-way table presents the number of subjects by whether RCLA holds and whether ambiguity neutrality holds. Each cell indicates the number of subjects with the expected number displayed in parentheses.

TABLE C.VIII Comparison between ambiguous and compound two-point lotteries.

Panel A: Ambiguity first

| | | $\{0,100\}$ | | | |
|---|---|---|---|---|---|
| | | $CE^A < CE^C$ | $CE^A = CE^C$ | $CE^A > CE^C$ | Total |
| {25,75} | $CE^A < CE^C$ | 7 (3.8) | 5 (5.8) | 3 (5.4) | 15 |
| | $CE^A = CE^C$ | 7 (11) | 19 (17.1) | 18 (15.9) | 44 |
| | $CE^A > CE^C$ | 4 (3.3) | 4 (5.1) | 5 (4.7) | 13 |
| | Total | 18 | 28 | 26 | 72 |

Panel B: Compound risk first

| | | $\{0,100\}$ | | | |
|---|---|---|---|---|---|
| | | $CE^A < CE^C$ | $CE^A = CE^C$ | $CE^A > CE^C$ | Total |
| {25,75} | $CE^A < CE^C$ | 8 (5.6) | 10 (9.2) | 0 (3.2) | 18 |
| | $CE^A = CE^C$ | 7 (8) | 12 (13.6) | 7 (4.7) | 26 |
| | $CE^A > CE^C$ | 2 (3.4) | 6 (5.6) | 3 (2) | 11 |
| | Total | 17 | 28 | 10 | 55 |

Note. The two-way table presents the number of subjects in each of the categories—$CE^A < CE^C$, $CE^A = CE^C$, and $CE^A > CE^C$—indicating that the CE of a two-point ambiguity lottery may be smaller than, equal to, or larger than that of the corresponding two-point compound lottery. Each cell indicates the number of subjects with the expected number displayed in parentheses.

TABLE C.IX Individual types with two-stage perspective.

Panel A: Ambiguity first

| | | Compound Risk | | | |
|---|---|---|---|---|---|
| | | REU | RRDU | Unclassified | Total |
| Ambiguity | REU | 11 (8.3) | 8 (0.2) | 4 (4.5) | 23 |
| | RRDU | 11 (14.1) | 21 (17.3) | 7 (7.6) | 39 |
| | Unclassified | 4 (3.6) | 3 (4.4) | 3 (1.9) | 10 |
| | Total | 26 | 32 | 14 | 72 |

Panel B: Compound risk first

| | | Compound Risk | | | |
|---|---|---|---|---|---|
| | | REU | RRDU | Unclassified | Total |
| Ambiguity | REU | 16 (12.3) | 5 (10.0) | 4 (2.7) | 25 |
| | RRDU | 6 (9.3) | 11 (7.6) | 2 (2.1) | 19 |
| | Unclassified | 5 (5.4) | 6 (4.4) | 0 (1.2) | 11 |
| | Total | 27 | 22 | 6 | 55 |

Note. The two-way table presents the number of subjects classified as REU or RRDU separately for ambiguity and compound risk. Each cell indicates the number of subjects with the expected number displayed in parentheses.

## C.2 Figures

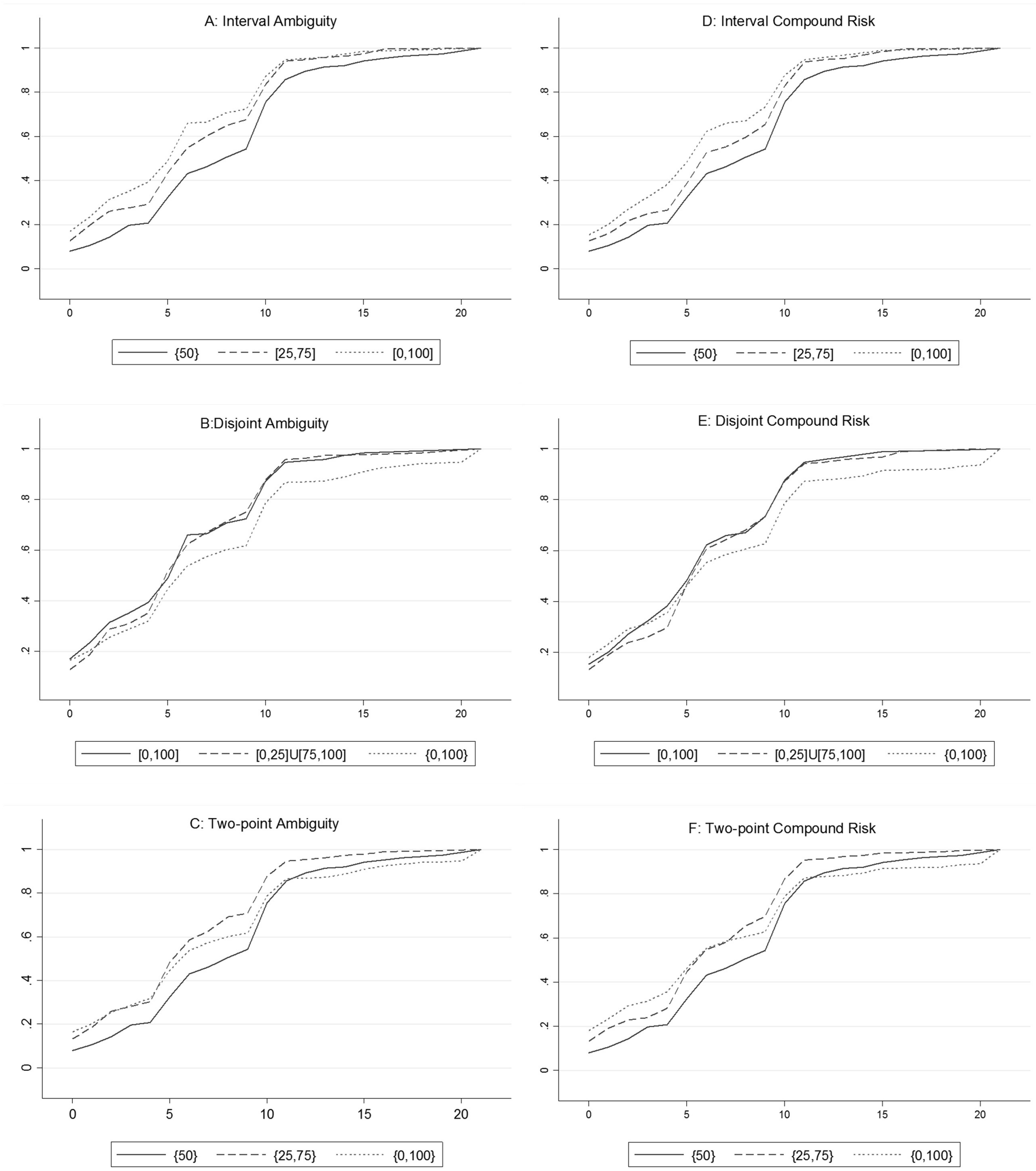



Figure C.1 Cumulative Distribution of CEs in each kind of partial ambiguity/compound risk.

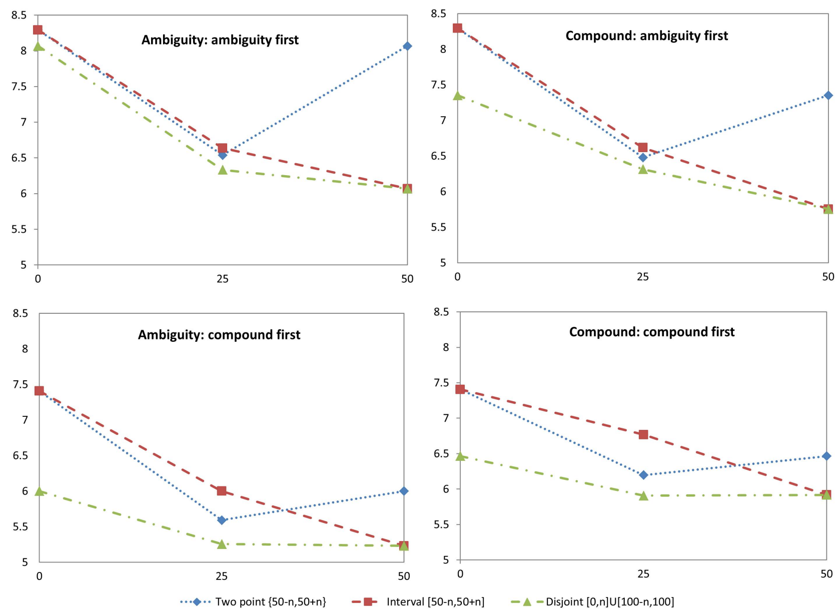



Figure C.2 Aggregate choice patterns for the two treatments.

### C.3 Order Effect

We assess the extent to which our results, especially those concerning the three key findings and individual type, are robust to the order of appearance of ambiguity and compound risk in our main experiment. The plots of the aggregate data displayed in Figure C.2 reveal that the observed choice patterns for interval, disjoint, and two-point ambiguity (compound risk) are generally similar across the two orders of appearance. Namely, there is a decreasing trend in the CEs for the interval and disjoint ambiguity (compound risk) as the number of possible compositions increases. For two-point ambiguity (compound risk), there is an initially decreasing trend in the CEs as the spread increases except for the end-point $\{0, 100\}$.

To check the robustness of the association between ambiguity attitude and RCLA for both orders, we plot the corresponding two-way tables separately (Table C.VII). For the 102 subjects under "ambiguity first", of the 19 subjects who are observed to reduce all compound lotteries, 18 are ambiguity neural. Of the 83 subjects who violate RCLA, 81 exhibit ambiguity non-neutrality. For the 86 subjects under "compound risk first", of the 12 subjects reducing all compound lotteries, all exhibit ambiguity neutrality. Of the 74 subjects who violate RCLA, 66 exhibit ambiguity non-neutrality. The null hypothesis of independence is strongly rejected in both order treatments (Pearson's chi-squared test, $p < 0.001$ for each treatment). This suggests that attitude towards partial ambiguity and attitude towards the corresponding compound risk are closely correlated within both treatments.

Table C.VIII further examines the robustness of the non-neutrality between two-point ambiguity and two-point compound risk. Under ambiguity first (compound risk first), 19 (12) subjects have similar CEs, 27 (16) subjects weakly prefer ambiguous two-point lotteries to compound two-point lotteries, and 19 (25) subjects exhibit the reverse behavior, with the rest of the 7 (2) subjects not revealing a consistent preference. The difference between the two treatments is significant for $\{0, 100\}$ (multinomial logistic regression, $p < 0.023$), but not for $\{25, 75\}$ (multinomial logistic regression, $p < 0.508$). Overall, while there are some differences between the two order treatments, the observation that substantial proportions of subjects value these two types of lotteries differently remains robust.

In an individual type analysis for REU and RRDU under the two treatments (see Table C.IX), 11 (16) are classified as REU type and 21 (11) are classified as RRDU type under ambiguity first (compound risk first) treatment. A proportion test reveals that the difference between these two treatments is statistically significant ($p < 0.039$). In sum, the three key findings appear robust to the two different orders. At the same time, we observe an order effect on the classification of individual type.

## Appendix D: Results of Supplementary Experiments

### D.1 Experiment S1 on Ambiguity

This subsection presents the design and results of Experiment S1 on partial ambiguity. We first present the experimental design, and then analyze the choice patterns for the three kinds of partial ambiguity at both aggregate and individual levels.

#### *D.1.1 Design*

Experiment S1 comprises three kinds of six lotteries each with a total of 15 lotteries as follows.
*Interval ambiguity.* It comprises six lotteries with interval ambiguity: $\{50\}$, $\mathbf{I}_{10}^A = [40, 60]^A$, $\mathbf{I}_{20}^A = [30, 70]^A$, $\mathbf{I}_{30}^A = [20, 80]^A$, $\mathbf{I}_{40}^A = [10, 90]^A$, $[0, 100]^A$.
*Disjoint ambiguity.* It involves six lotteries with disjoint ambiguity: $\{0, 100\}^A$, $\mathbf{D}_{10}^A = [0, 10] \cup [90, 100]^A$, $\mathbf{D}_{20}^A = [0, 20] \cup [80, 100]^A$, $\mathbf{D}_{30}^A = [0, 30] \cup [70, 100]^A$, $\mathbf{D}_{40}^A = [0, 40] \cup [60, 100]^A$, $[0, 100]^A$.
*Two-point ambiguity.* It involves six lotteries with two-point ambiguity: $\{50\}$, $\mathbf{T}_{10}^A = \{40, 60\}^A$, $\mathbf{T}_{20}^A = \{30, 70\}^A$, $\mathbf{T}_{30}^A = \{20, 80\}^A$, $\mathbf{T}_{40}^A = \{10, 90\}^A$, $\{0, 100\}^A$.

In the experiment, subjects were shown the 15 decks of cards. For each lottery, betting correctly on the color of a drawn card would deliver SGD40 (about USD30) while betting incorrectly would deliver nothing. We use the price-list design and RIM in eliciting the CEs of various lotteries. The order of appearance of the 15 lotteries is randomized for the subjects who each make 150 choices in total (see Appendix E for detailed Experimental instructions). At the end of the experiment, in addition to a SGD5 show-up fee, each subjects is paid based on his/her randomly selected decision in the experiment. One out of 150 choices is randomly chosen using dice. We recruited 112 undergraduate students from the National University of Singapore (NUS) by advertising on the university platform, the Integrated Virtual Learning Environment. The experiment consisted of four sessions with 20 to 30 subjects in each session.

#### *D.1.2 Observed Choice Behavior*

We present the observed choice behavior at both aggregate and individual levels for 106 subjects. Six subjects exhibit multiple switching in some of the tasks. Their data are excluded from our analysis. The choice data are similarly coded in terms of the switch point given by the number of times a subject chooses a given lottery over different increasingly ordered sure amounts before switching over to choosing the sure amounts.

We first examine the implication of ambiguity neutrality, i.e., that subjects assign the same CE to the 15 lotteries. Using a Friedman test ($p < 0.001$), we reject the null hypothesis that the CEs of the 15 lotteries come from a single distribution. Besides replicating the standard finding on ambiguity aversion with CE of $\{50\}$ being significantly higher than that of $[0, 100]^A$ (paired Wilcoxon Signed-rank test, $p < 0.001$), our subjects have distinct attitudes towards different kinds of partial ambiguity. Specifically, for the comparison between $\{50\}$ and $[0, 100]^A$, 62 subjects (58.5%) exhibit ambiguity aversion, 33 subjects (31.1%) exhibit ambiguity neutrality, and 11 subjects (10.4%) exhibit ambiguity affinity. Comparing $\{50\}$ with the other 14 ambiguous lotteries at the individual level, 16 out of 106 subjects (15.1%) have the same CEs for the 15 lotteries. Among the others, 48 subjects (45.3%) exhibit overall ambiguity aversion in having weakly larger CEs for $\{50\}$ than for the other 14 ambiguous lotteries. Thirteen subjects (12.3%) have weakly lower CEs for $\{50\}$ than that for the other 14 ambiguous lotteries, hence revealing some degree of ambiguity affinity. The remaining 29 subjects (24.3%) do not exhibit uniform attitude towards ambiguity. Using a similar analysis as in the main experiment, we replicate the observations for the three kinds of partial ambiguity.

For interval ambiguity, there is a statistically significant *decreasing* trend in the CEs as the number of possible compositions increases ($p < 0.001$). At the individual level, 20 subjects (18.9%) have the same CEs, 25 subjects (23.6%) have weakly decreasing CEs, while none of the subjects has weakly increasing CEs. For disjoint ambiguity, there is a statistically significant *decreasing* trend in the CEs as the number of possible compositions increases ($p < 0.001$). At the individual level, 19 subjects (17.9%) have the same CEs, 19 subjects (17.9%) have weakly decreasing CEs, and four subjects (3.8%) have weakly increasing CEs.

For two-point ambiguity, there is a significant *decreasing* trend in the CEs until the end point. Interestingly, the CE of $\{0, 100\}^A$ reverses this trend and is significantly higher than the CE of $\{10, 90\}^A$ (paired Wilcoxon Signed-rank test, $p < 0.001$). Moreover, the CE of $\{10, 90\}^A$ is not significantly different from that of $\{50\}$ (paired Wilcoxon Signed-rank test, $p > 0.225$). At the individual level, 22 subjects (20.8%) have the same CEs, 14 subjects (13.2%) have weakly decreasing CEs, 33 subjects (31.1%) have weakly decreasing CEs until $\{10, 90\}^A$ with an increase at $\{0, 100\}^A$, and seven subjects (6.6%) have weakly increasing CEs. Between $\{50\}$ and $\{0, 100\}^A$, 49 subjects (46.2%) have the same CEs, 33 subjects (31.1%) display a higher CE for $\{50\}$ than that for $\{0, 100\}^A$, and 24 subjects (22.6%) exhibit the reverse.

In summary, the observed patterns towards the three kinds of ambiguity replicate the observations in the main experiment regarding ambiguity lotteries.

### D.2 Experiment S2 on Compound Risk

This subsection presents the design and results of Experiment S2 on compound lottery. We first present the experimental design, and then analyze the choice patterns for the three kinds of compound lottery at both aggregate and individual levels.

*D.2.1 Design*

Experiment S2 on uniform compound risk links naturally to the two-stage perspective of ambiguity under uniform priors. For each compound lottery, we implement objective uniform stage-1 prior as follows: one ticket is randomly drawn from a bag containing some tickets with different numbers written on them. The number drawn determines the number of red cards in

the deck with the rest black. The resulting stage-1 risk is thus uniformly distributed among different numbers while the bet at stage 2 involves betting on the color of a card randomly drawn from the deck. There are three kinds of nine lotteries included in this experiment.
*Interval compound risk.* This involves four lotteries with symmetric interval stage-1 risk: $\{50\}$, $\mathbf{I}_{10}^C = [40, 60]^C$, $\mathbf{I}_{30}^C = [20, 80]^C$, $[0, 100]^C$.
*Disjoint compound risk.* This involves four lotteries with symmetric disjoint stage-1 risk: $\{0, 100\}^C$ , $\mathbf{D}_{10}^C = [0, 20] \cup [80, 100]^C$, $\mathbf{D}_{30}^C = [0, 40] \cup [60, 100]^C$, $[0, 100]^C$.
*Two-point compound risk.* This involves four lotteries with symmetric two-point stage-1 risk: $\{50\}$, $\mathbf{T}_{10}^C = \{40, 60\}^C$, $\mathbf{T}_{30}^C = \{20, 80\}^C$, $\{0, 100\}^C$ .

The elicitation mechanism and experimental procedure are similar as in Experiment S1 (see Appendix E for detailed Experimental instructions). We have 109 subjects in Experiment S2.

*D.2.2 Observed Choice Behavior*

We report the observed choice patterns at both aggregate and individual levels. To test the implication of RCLA, i.e., that the CEs of the 9 lotteries are the same, we apply the Friedman test and reject the null hypothesis that their CEs come from the same distribution ($p < 0.001$).At the individual level, 12 of 98 subjects (12.2%) have the same CE for the 9 lotteries. Besides these subjects, 39 subjects (40.0%) have weakly larger CEs for $\{50\}$ than that of the other eight compound lotteries while 17 subjects (17.3%) exhibit the opposite pattern with weakly lower CEs for $\{50\}$ than the other eight lotteries. This suggests that subjects tend to weakly prefer receiving the reduced simple lottery than any of the eight other compound lotteries. To study the choice patterns across the three kinds of compound lotteries, we again apply a similar analysis as in the main experiment to examine whether there is a significant trend in each kind corresponding to attitudes towards different patterns of spread in stage-1 risks.

For interval compound risk, there is a statistically significant *decreasing* trend in the CEs as the stage-1 risks spread away from the mid-point ($p < 0.001$). At the individual level, 21 subjects (21.4%) have the same CEs, 30 subjects (30.6%) have weakly increasing CEs, and 12 subjects (12.2%) have weakly decreasing CEs, with the rest of the subjects not exhibiting monotonic preference in relation to uniform interval spread. For disjoint compound risk, there is a statistically significant *increasing* trend in the CEs as the stage-1 risks spread away from the mid-point ($p < 0.001$). At the individual level, 16 subjects (16.3%) have the same CEs, 26 subjects (26.5%) have weakly increasing CEs, and 22 subjects (22.4%) have weakly decreasing CEs, with the rest of the subjects not exhibiting monotonic preference in relation to uniform disjoint spread.

For two-point compound risk, there is a statistically significant *decreasing* trend in the CEs as the stage-1 risks spread away from the mid-point ($p < 0.042$). At the individual level, 18 subjects (18.4%) have weakly increasing CEs, 14 subjects (14.3%) have the same CEs, 8 subjects (8.2%) have weakly decreasing CEs, and 17 subjects (17.3%) have weakly decreasing CEs initially followed by an increase near the end point, while the remaining 41 subjects (42.0%) do not exhibit any of these patterns. Focusing on $\{50\}$ and $\{0, 100\}^C$, 34 subjects (34.7%) have the same CEs, 43 subjects (43.9%) have a higher CE for $\{50\}$ than that of $\{0, 100\}^C$, while the other 21 subjects (21.4%) have the reverse preference. Paired Wilcoxon

Signed-rank test shows that the CE of $\{50\}$ is significantly higher than that of $\{0, 100\}^C$($p < 0.022$).

In summary, the observed patterns towards the three kinds of compound lottery replicate the observations in the main experiment except for the end-point, which we elaborate in the manuscript.

# REFERENCES


[1] Abdellaoui, M., A. Baillon, L. Placido, and P. P. Wakker (2011): "The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation," *American Economic Review* 101: 695-723.

[2] Chew, S. H., Epstein, L. G., and U. Segal (1991): "Mixture Symmetry and Quadratic Utility," *Econometrica* 59: 139-163.

[3] Chew, S. H., B. Miao., and S. Zhong (2013): "Partial Ambiguity," *working paper, http://papers.ssrn.com/sol3/papers.cfm?abstractid=2225580.*

[4] Chew, S. H., and J. S. Sagi (2008): "Small Worlds: Modeling Preference over Sources of Uncertainty," *Journal of Economic Theory* 139: 1-24.

[5] Gajdos, T., T. Hayashi., J. M. Tallon., and J. C. Vergnaud (2008): "Attitude toward Imprecise Information," *Journal of Economic Theory* 140: 23–56.

[6] Ghirardato, P., F. Maccheroni, and M. Marinacci (2004): "Differentiating Ambiguity and Ambiguity Attitude," *Journal of Economic Theory* 118: 133–173.

[7] Gilboa, I., and D. Schmeidler (1989): "Maxmin expected Utility with Non-unique Prior," *Journal of Mathematical Economics* 18: 141-153.

[8] Maccheroni, F., M. Marinacci, and A. Rustichini (2006): "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences," *Econometrica* 74: 1447-1498.

[9] Siniscalchi, M. (2009): "Vector Expected Utility and Attitudes toward Variation," *Econometrica* 77: 801-855.

# Appendix E: Experimental Instructions

<u>**GENERAL INSTRUCTIONS (Main Experiment)**</u>

Welcome to our study on decision making. The descriptions of the study contained in this instrument will be implemented **fully.**

**Each participant will receive on average $20 for the study. The overall compensation includes a $5 show up fee in addition to earnings based on how you make decisions.**

**All information provided will be kept CONFIDENTIAL.** Information in the study will be used for research purposes only. Please refrain from discussing any aspect of the specific tasks of the study with any one.

1. **It is important to read the instructions CAREFULLY** so that you understand the tasks in making your decisions.
2. **PLEASE DO NOT communicate** with others during the experiment.
3. Cell phones and other electronic communication devices are **not allowed.**
4. **At ANY TIME,** if you have questions, please raise your hand**.**

The study consists of 11 decision sheets of the form illustrated below.

| | Option A | Option B | Decision |
|---|---|---|---|
| *1* | Betting on the cards | Receiving $8 for sure | A ☐ B ☐ |
| *2* | Betting on the cards | Receiving $10 for sure | A ☐ B ☐ |
| *3* | Betting on the cards | Receiving $12 for sure | A ☐ B ☐ |
| *4* | Betting on the cards | Receiving $13 for sure | A ☐ B ☐ |
| *5* | Betting on the cards | Receiving $14 for sure | A ☐ B ☐ |
| *6* | Betting on the cards | Receiving $15 for sure | A ☐ B ☐ |
| *7* | Betting on the cards | Receiving $16 for sure | A ☐ B ☐ |
| *8* | Betting on the cards | Receiving $17 for sure | A ☐ B ☐ |
| *9* | Betting on the cards | Receiving $18 for sure | A ☐ B ☐ |
| *10* | Betting on the cards | Receiving $19 for sure | A ☐ B ☐ |
| *11* | Betting on the cards | Receiving $20 for sure | A ☐ B ☐ |
| *12* | Betting on the cards | Receiving $21 for sure | A ☐ B ☐ |
| *13* | Betting on the cards | Receiving $22 for sure | A ☐ B ☐ |
| *14* | Betting on the cards | Receiving $23 for sure | A ☐ B ☐ |
| *15* | Betting on the cards | Receiving $24 for sure | A ☐ B ☐ |
| *16* | Betting on the cards | Receiving $25 for sure | A ☐ B ☐ |
| *17* | Betting on the cards | Receiving $26 for sure | A ☐ B ☐ |
| *18* | Betting on the cards | Receiving $27 for sure | A ☐ B ☐ |
| *19* | Betting on the cards | Receiving $28 for sure | A ☐ B ☐ |
| *20* | Betting on the cards | Receiving $30 for sure | A ☐ B ☐ |
| *21* | Betting on the cards | Receiving $32 for sure | A ☐ B ☐ |

Each such table lists <u>21 choices</u> to be made between a fixed Option A and 21 different Option B's.

**Option A** involves a **lottery**, guessing the color of a card randomly drawn from a deck of 100 cards with different compositions of red and black. If you guess correctly, you receive $40; otherwise you receive nothing. **Different tasks will have different compositions of red and black cards as described in each task.**

The Option B's refer to receiving the specific amounts of money for sure, and are arranged in an <u>ascending manner in the amount of money from $8 to $32</u>.

For each row, you are asked to indicate your choice in the final "Decision" column – A or B – either with a tick (√) or by drawing vertical lines as indicated above.

**<u>Selection of decision sheet to be implemented</u>**: **One out of the 11** Decision Sheets (*selected randomly by you*) will be implemented. Should the sheet be chosen, **one of your 21 choices** will be further selected randomly and implemented.

**Part I**

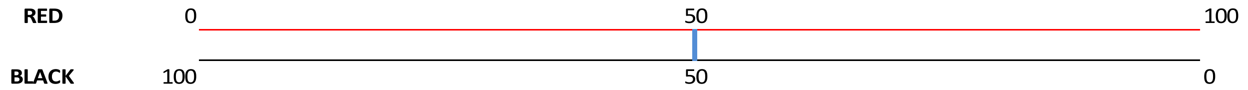
This is Part I, which has 6 decision sheets with different composition of cards. The description of each of the 6 bets begins with:

**Option A**: You guess the color first. You then draw a card from the deck of cards constructed in the manner described below**. If you guess the color correctly, you receive $40**. **Otherwise, you receive $0.**

This is followed by specific wordings for the 6 bets which are provided below. We begin with a bet which is based on a 100-card deck whose composition is known: 50 red cards and 50 black cards.

**Bet I {50}**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards is 50 and the number of black cards is also 50, as illustrated below.




For the remaining 5 bets, the composition of the cards and procedure of the bets are described as follows.

**Stage 1.** You guess the color of a card to be drawn from a deck to be constructed in Stage 2.

**Stage 2.** You draw one ticket from an **envelope** containing tickets of different numbers. The number drawn will determine the number of red cards in a deck of 100 cards. If the ticket drawn is 30, then the deck will have 30 red card and 70 black cards. If the ticket drawn is 68, then the deck will have 68 red cards and 32 black cards. **Note that different bets will involve different compositions of numbers in the envelope.**

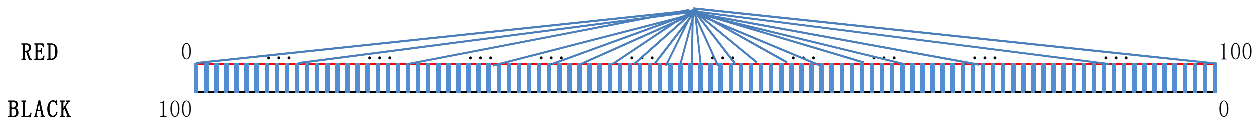
**Stage 3. Y**ou will draw randomly one card from the 100 cards as constructed in stage 2 and receive $40 if your guess in Stage 1 is correct.

*Summary: Guess the color in Stage 1. Draw a numbered ticket randomly to construct the deck in Stage 2. Then draw a card in Stage 3 to see if you have guessed correctly.*

The 5 bets are explained below.

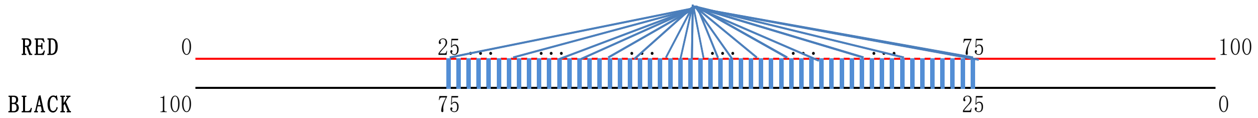
**Bet I [0-100]**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The composition of this card deck is determined as follows. You draw one ticket from an envelope containing 101 tickets numbered from 0 and 100. The number you draw determines how many red cards there are in this deck of 100 cards.



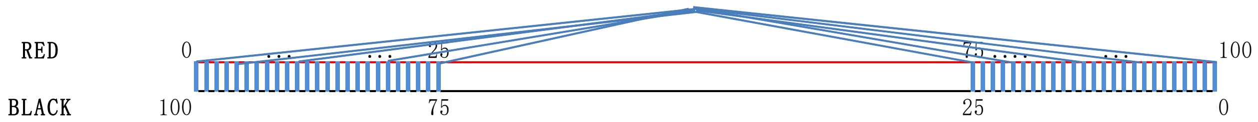
**Bet I [25-75]**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The composition of this card deck is determined as follows. You draw one ticket from an envelope containing 51 tickets numbered from 25 and 75. The number you draw determines how many red cards there are in this deck of 100 cards.



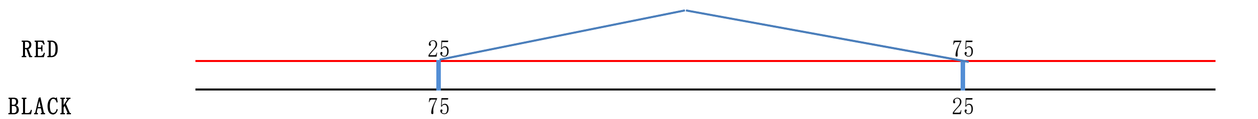

**Bet I [0-25]U[75-100]**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The composition of this card deck is determined as follows. You draw one ticket from an envelope containing 52 tickets numbered from 0 and 25 and 75 to 100. The number you draw determines how many red cards there are in this deck of 100 cards.



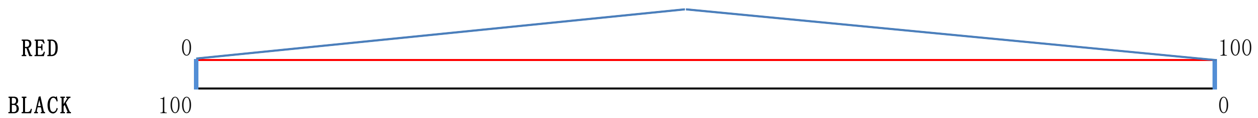

**Bet I {25, 75}**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The composition of this card deck is determined as follows. You draw one ticket from an envelope containing only two tickets numbered 25 and 75. The number you draw determines how many red cards there are in this deck of 100 cards.




**Bet I {0, 100}**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The composition of this card deck is determined as follows. You draw one ticket from an envelope containing only two tickets numbered 0 and 100. The number you draw determines how many red cards there are in this deck of 100 cards.

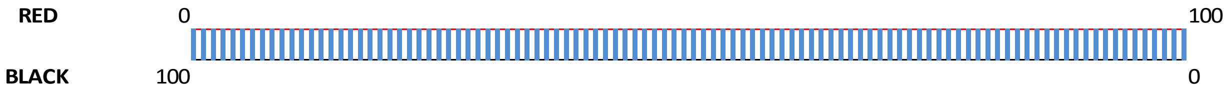


**Part II**

This is Part II, which consists of 5 decision sheets similiar as that in Part I. The composition of the cards are unknown as described below.

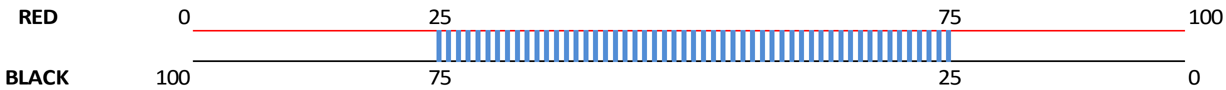
**Bet II [0-100]**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 100 with the rest of the cards black, as illustrated below.



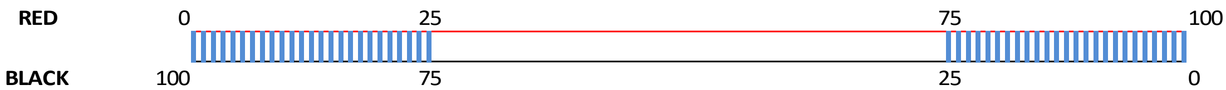

**Bet II [25-75]**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 25 to 75 with the rest of the cards black, as illustrated below.



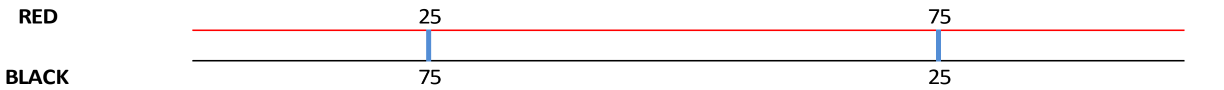

**Bet II [0-25]U[75-100]**

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 25 or from 75 to 100 with the rest of the cards black, as illustrated below.



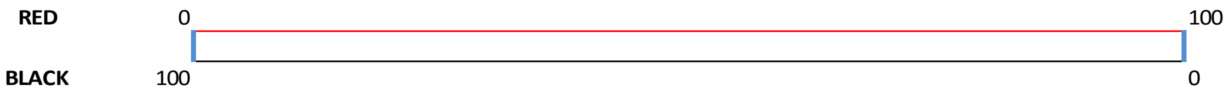

**Bet II {25, 75}**

This situation involves your drawing a card randomly from a deck of 100 cards containing either 25 or 75 red cards with the rest of the cards black, as illustrated below.




**Bet II {0, 100}**

This situation involves your drawing a card randomly from a deck of 100 cards containing either 100 red cards or 100 black cards, as illustrated below.




[This is the instructions for compound first treatment. The instructions for ambiguity first treatment are presented in a similar manner.]

## GENERAL INSTRUCTIONS (Experiment S1)

Welcome to our study on decision making. The descriptions of the study contained in this instrument will be implemented **fully and faithfully**.

**Each participant will receive on average $20 for the study. The overall compensation includes a $5 show up fee in addition to earnings based on how you make decisions.**

**All information provided will be kept CONFIDENTIAL.** Information in the study will be used for research purposes only. Please refrain from discussing any aspect of the specific tasks of the study with any one.

**1.** The set of decision making tasks and the instructions for each task are **the same** for all participants.
**2.** **It is important to read the instructions CAREFULLY** so that you understand the tasks in making your decisions.
**3.** If you have questions, please raise your hand to **ask our experimenters at ANY TIME.**
**4.** **PLEASE DO NOT communicate** with others during the experiment.
**5.** Do take the time to go through the instructions carefully in making your decisions.
**6.** Cell phones and other electronic communication devices are **not allowed.**

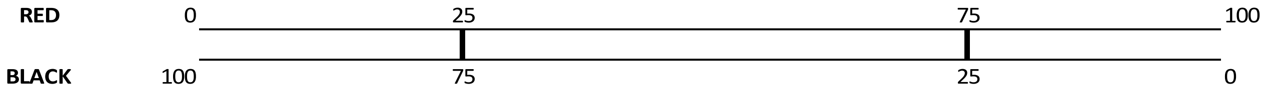
This is the first part for today's study comprising **15 decision sheets** each of which is of the form illustrated in the table below.

| | Option A | Option B | Decision |
|---|---|---|---|
| 1 | A | B1 | A □ B □ |
| 2 | A | B2 | A □ B □ |
| 3 | A | B3 | A □ B □ |
| 4 | A | B4 | A □ B □ |
| 5 | A | B5 | A □ B □ |
| 6 | A | B6 | A □ B □ |
| 7 | A | B7 | A □ B □ |
| 8 | A | B8 | A □ B □ |
| 9 | A | B9 | A □ B □ |
| 10 | A | B10 | A □ B □ |

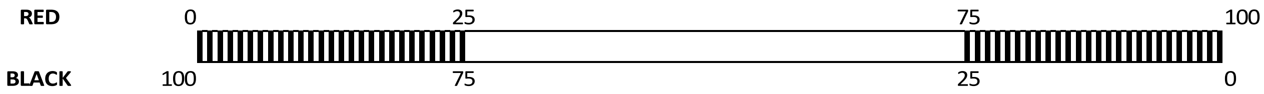
Each such table lists **10 choices** to be made between a fixed **Option A** and 10 different **Option B**'s.

**Option A** involves a **lottery**, guessing the color of a card randomly drawn from a deck of 100 cards with different compositions of red and black. If you guess correctly, you receive $40; otherwise you receive nothing. **Different tasks will have different compositions of red and black cards as described for each task.**

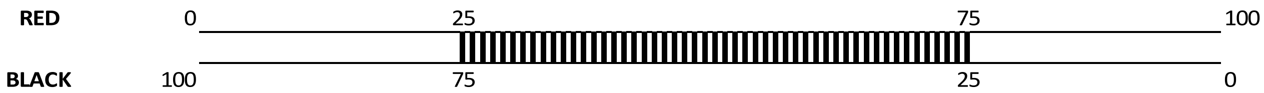
*Example 1: This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The deck, illustrated below, has* ***either*** *25* ***or*** *75 red cards with the rest of the cards black.*




*Example 2: This situation involves your drawing a card randomly from a deck of 100 cards being made up of red and black cards. The number of red cards, illustrated below, may be anywhere between 0 and 25 or between 75 and 100 with the rest black.*




*Example 3: This situation involves your drawing a card randomly from a deck of 100 cards made up of red and black cards. The number of red cards, illustrated below, may be anywhere between 25 and 75 with the rest of the cards black.*




**The Option B's** refer to receiving the specific amounts of money for sure, and are arranged in an **<u>ascending manner in the amount of money</u>.**

For each row, you are asked to **<u>indicate your choice</u>** in the final "Decision" column – A or B – with a tick **(√).**

**<u>Selection of decision sheet to be implemented</u>**: **One out of the 15** Decision Sheets (*selected randomly by you*) will be implemented. Should the sheet be chosen, **one of your 10 choices** will be further selected randomly and implemented.

## GENERAL INSTRUCTIONS (Experiment S2)

Welcome to our study on decision making. The descriptions of the study contained in this experimental instrument will be implemented **fully and faithfully**.

**Each participant will receive on average $20 for the study. The overall compensation includes a $5 show up fee in addition to earnings based on how you make decisions.**

**All information provided will be kept CONFIDENTIAL.** Information in the study will be used for research purposes only. You are not to discuss with anyone any aspect of the specific tasks during or after the study.

**1.** The set of decision making tasks and the instructions for each task are **the same** for all participants.
**2.** **It is important to read the instructions CAREFULLY** so that you understand the tasks in making your decisions.
**3.** If you have questions, please raise your hand to **ask our experimenters at ANY TIME.**
**4.** **PLEASE DO NOT communicate** with others during the experiment.
**5.** Do take the time to go through the instructions carefully in making your decisions.
**6.** Cell phones and other electronic communication devices are **not allowed.**

The experiment comprises **9 decision sheets**, which are of the form illustrated in the table below.

| | Option A | Option B | Decision |
|---|---|---|---|
| 1 | A | B1 | A □ B □ |
| 2 | A | B2 | A □ B □ |
| 3 | A | B3 | A □ B □ |
| 4 | A | B4 | A □ B □ |
| 5 | A | B5 | A □ B □ |
| 6 | A | B6 | A □ B □ |
| 7 | A | B7 | A □ B □ |
| 8 | A | B8 | A □ B □ |
| 9 | A | B9 | A □ B □ |
| 10 | A | B10 | A □ B □ |

Each such table lists **10 choices** to be made between a fixed **Option A** and 10 different **Option B's**.

**Option A** involves **lotteries**, guessing the color of card randomly drawn from a deck of 100 cards with different proportions of red and black. If you guess correctly, you receive $40; otherwise you receive nothing.

In stage 1, You guess the color. The proportion of red and black is determined in the stage 2 as follows.

In stage 2, You draw one ticket from an envelope containing some tickets with different numbers. The number you draw determines how many red cards there are in those 100 cards. *If the ticket drawn is 0, then the deck will have 0 red card and 100 black cards. If the ticket drawn is 100, then the deck will have 100 red cards and 0 black cards. If the ticket drawn is 50, then the deck will have 50 red cards and 50 black cards.* **Different task will have different composition of numbers in the envelop.**

In stage 3, you will draw randomly one card from the 100 cards as constructed in stage 2 and if your initial guess is correct, you will receive $40.

*For example, you may guess a color (either red or black) and then draw a number randomly from 30-70. If the number drawn is 45, this means that the deck will consist of 45 red cards and 55 black cards. Finally, you draw a card randomly from the deck, and you receive $40 if your guess is correct, otherwise you receive nothing.*

**Option B**'s a certain amount of money you can choose to receive, and it is arranged in an **ascending manner in the amount of money.**

For each row, you are asked to **indicate your choice** in the final "Decision" column – A or B – with a tick (√).

**Selection of decision sheet to be implemented**: **One out of the 9** Decision Sheets (*selected randomly by you*) will be implemented. Should the sheet be chosen, **one of your 10 choices** will be further selected randomly and implemented.

You may now begin.