

Ellsberg meets Keynes at an Urn*

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Abstract

Ellsberg and Keynes have articulated independently a preference for betting on the color of a ball drawn from an urn with a known 50-50 composition of two colors to betting on another urn with an unknown composition of the two colors. Ellsberg describes this as ambiguity aversion in terms of weighing the possible priors pessimistically. By contrast, Keynes views this as preference between two sources of uncertainty both involving 50-50 bets while differing in terms of weight of evidence. In an experiment, we study attitudes towards ambiguity, compound risk and almost-objective uncertainty and find that two-thirds of the subjects are non-neutral towards all three kinds of uncertainty. Moreover, we observe that ambiguity attitude and compound risk attitude are individually linked to attitude towards almost-objective uncertainty, in addition to replicating the known association between attitudes toward ambiguity and objective compound lottery, motivating the need for further theoretical development.

Keywords: Ambiguity, Source preference, Maxmin expected utility, Recursive utility, Experiment

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1 Introduction

In *A Treatise on Probability* (1921), Keynes articulates “*if two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge*”. To illustrate this point, he proposes a thought experiment: between betting on an urn with a known composition of 50 red and 50 black balls over betting on another urn with an unknown composition of red and black balls which sum to 100, and points out that the probability of drawing a red (black) ball from either urn is evidently $1/2$, but the “weight of argument” is greater for the known urn leading possibly to a preference for the known bet. Beyond Keynes’ two-urn example, Fox and Tversky (1995) consider uncertainty arising from natural events such as the price of a stock and temperature of a city. In their experiment, subjects in the Bay area value separately two pairs of bets with potentially similar likelihoods of winning: one pair is based on whether the temperature one week from today in San Francisco is above or below 60°F ; another pair corresponds to betting on the temperature in Istanbul also using 60°F . Surprisingly, they observe that the lower-valued bet on San Francisco temperature is still worth more than that of the higher-valued bet on Istanbul temperature. Fox and Tversky originate the concept of source preference referring to a preference for betting on sources of uncertainty, such as a known urn or a familiar city involving greater knowledge.

Echoing Keynes, Smith (1969) arrives independently at the view that both the known and unknown bets are in effect 50-50 when subjects do not prefer one color over the other for balls drawn from either urn. He proposes to make use of two distinct vNM utility functions so that a known bet can be worth more than an unknown bet. This can be accomplished as long as the utility function associated with risks arising from the unknown urn is more concave than that associated with risks from the known urn.¹ Naturally, this idea can be extended to model familiarity preference with a less concave vNM utility function being associated with the more familiar source of uncertainty. This source preference perspective prompts Chew and Sagi (2008) to offer an axiomatization more generally so that the decision maker can have distinct attitudes towards risks arising from different sources of uncertainty that induce the same distribution, whether they are from an urn, from a deck of cards, or from natural events such as the temperature of a city.

The preference for the known bet over the unknown bet is later independently discovered by Ellsberg (1961) who labels it as ambiguity aversion. He observes that the unknown bet

¹Smith (1969) attempts to model preference for the known urn bets by arguing that the “known” utility is cardinally higher than the “unknown” utility. It is more efficient for him to apply Pratt’s (1964) comparative risk aversion result on the two distinct vNM utility functions to show that bets from the known urn can always exceed bets from the unknown urn.

encompasses a range of possible priors, and hypothesizes that ambiguity aversion occurs for those who are pessimistic in weighing the less favorable priors disproportionately. This multiple-prior perspective gives rise to two directions in modeling ambiguity attitude. One direction is to assign all weights to the worst prior as axiomatized in Gilboa and Schmeidler’s (1989) model which delivers the lowest among the possible expected utilities.² A parallel direction views ambiguity as a distribution over the range of possible priors (Becker and Brownson, 1964), through which the unknown bet delivers a subjective compound lottery that can be reduced to an even-chance simple lottery. As such, ambiguity aversion corresponds a preference for the even-chance simple lottery from the known urn over the induced subjective compound lottery from the unknown urn, thereby violating the reduction of compound lottery axiom (RCLA).³ Adopting this view, Segal (1987) models ambiguity aversion by applying a probability weighting function that overweighs the less favored priors. This theoretical relation between the two views of ambiguity involving multiple priors motivates Halevy’s (2007) experimental study the link between attitudes towards ambiguity and objective compound lottery, in which the distribution of priors is objectively given as uniform. Halevy (2007)’s finding of a tight link between these two attitudes lends support to the compound lottery view and the corresponding recursive modeling approach of Segal (1987) and subsequently Klibanoff, Marinacci, and Mukerji (2005), Ergin and Gul (2009), and Seo (2009).

While the source preference approach encompasses distinct attitudes toward a wide range of uncertainty, including known and unknown urns as well as those arising from natural events such as the temperature of a city, little is known about the possible links among attitudes towards different types of uncertainty. Building on recent experimental studies (e.g., Halevy, 2007; Abdellaoui, Klibanoff and Placido, 2015; Chew, Miao and Zhong, 2017) regarding the link between ambiguity attitude and compound risk attitude, this study aims to examine how the two attitudes may relate individually to attitude towards natural-event uncertainty. In our experiment, besides the benchmark 50-50 known risk based on betting on the color of a card drawn from a deck of 20 cards with 10 red and 10 black, we consider three types of uncertainty:

Ambiguity: Betting on the color of a card drawn from a deck of 20 cards each being either red or black. The composition is (partially) unknown.

²Ghirardato, Maccheroni, and Marinacci (2004) offer a more moderate model of pessimism through a convex combination of the best and the worst prior. Gajdos et al. (2008) axiomatize the pessimism decision rule in Ellsberg (1961) which takes a convex combination of the worst prior and an “estimated” half/half prior. Schmeidler (1989) and Gilboa (1987) can capture the intuition of pessimism directly by assigning greater decision weight than 1/2 to losing event.

³RCLA requires that a compound lottery is indifferent to its reduction to a simple lottery with the same overall probabilities and outcomes.

Compound risk: Betting on the color of a card drawn from a deck of 20 cards each being either red or black. The composition is objectively determined by a random device.

Natural-event uncertainty: Betting on the trailing decimal digit (odd versus even) of the highest temperature in a city on a randomly selected historical date.

We include three ambiguity lotteries, two compound lotteries, and two natural-event lotteries based on two different cities. All these lotteries deliver a half chance of winning in the spirit of Keynes. Notice that our choice of natural-event uncertainty is distinct from that in Fox and Tversky (1995) in that the events of odd/even in the temperature bets are 50-50 in an ‘almost objective’ sense (see Section III for a detailed discussion). We adopt a within-subject design with more than 2000 undergraduate subjects in Singapore. Using a choice-list mechanism, we elicit the certainty equivalent (CE) for each lottery.

We infer subjects’ attitude towards ambiguity (compound risk/natural event) by its premium, which is given by the difference between the CE of the benchmark 50-50 risky lottery and that of ambiguity (compound risk/natural event) lottery. In other words, there is neutrality (aversion) towards a certain type of uncertainty when the corresponding premium is zero (positive). The majority of our subjects exhibit distinct attitudes toward the three types of uncertainty. Three-fourths of the subjects exhibit non-neutrality towards ambiguity as well as compound risk and two-thirds exhibit non-neutrality towards the natural-event uncertainty of a city temperature. Besides replicating the significant association between ambiguity attitude and compound risk attitude in the literature, we find that natural-event uncertainty attitude is closely linked to both attitudes toward ambiguity and compound risk. Subjects who are neutral towards the odd-even bet on temperature are more likely to be ambiguity neutral and more likely to reduce compound lottery. We observe significant correlations for premium across the three types of uncertainty and, as anticipated, the correlation coefficient between ambiguity and compound risk is substantially higher than the other two correlation coefficients (Table I).

Table I
Correlations Across Three Types of Uncertainty

	Ambiguity	Compound Risk
Compound Risk	0.823 (0.014)	
Natural Event	0.563 (0.009)	0.657 (0.011)

Note: The correlation is estimated using the Obviously Related Instrumental Variables (Gillen, Snowberg and Yariv, 2018) with standard errors.

In individual type analysis, we find that 66.0% of the subjects are non-neutral towards all three types of uncertainty, and 5.7% of the subjects are neutral towards all three. For the rest of subjects, 20.4% are non-neutral towards ambiguity and compound risk, and are neutral towards natural-event uncertainty. These three groups of subjects account for 92.1% of the subjects, with the rest of 7.9% exhibit other behavioral patterns. Moreover, the observed patterns do not differ substantially between subjects with different high and low cognitive ability measured by Raven’s Progressive Matrices. Finally, we show that the observed results are also robust with respect to replication using greater than 1000 undergraduate subjects in Beijing.

Our results shed light on decision theoretic models under uncertainty. In particular, our inclusion of natural-event uncertainty helps distinguish models through an orthogonal perspective compared to Halevy (2007) among others. To elaborate, the link between ambiguity attitude and compound risk attitude found in this latter stream of literature, replicated in the present paper, favors recursive formulations such as Segal (1987) along with Klibanoff, Marinacci, and Mukerji (2005), Nau (2006), Ergin and Gul (2009) and Seo (2009) which do not require RCLA against models which assume RCLA, e.g., Gilboa and Schmeidler (1989) and Chew and Sagi (2008). At the same time, where natural-event uncertainty is concerned, the observed non-neutrality towards almost objective events points to the need to incorporate source preference in distinguishing among identically distributed risks. Together with the observed close links between attitude towards natural-event uncertainty and ambiguity attitudes as well as compound risk attitude, our overall findings point to the need to extend the current recursive models by incorporating within-stage source preference.

Our results are also related to recent studies linking ambiguity attitudes in the lab and choice behavior in the real life. Warnick, Escobal and Laszlo (2011) experimentally measure ambiguity attitudes of farmers in rural Peru and observe that ambiguity aversion is negatively correlated with adoption of new technologies in the sense of planting less variety of main crop. Dimmock et al. (2016) show that ambiguity aversion in the Ellsberg urn is negatively associated with stock market participation, the fraction of financial assets in stocks, and foreign stock ownership, and positively related to own-company stock ownership. Dimmock, Kouwenberg and Wakker (2015) measure ambiguity attitudes using probability matching and show that ambiguity aversion is negatively related to stock market participation for subjects who perceive stock returns as highly ambiguous. Under our analytical framework linking attitudes toward different types of uncertainty, these studies are consistent with the explanation that attitudes towards uncertainty in the lab are related to those in real life.

The rest of this paper is organized as follows. Section 2 presents the overall experimental design. Section 3 reviews the theoretical literature of decision making under uncertainty, and

derives their implications for attitudes towards different types of uncertainty. The observed patterns are reported in Section 4 including summary statistics, correlation across types of uncertainty, and individual type analysis. Section 5 discusses the implications of our findings. We conclude in Section 6.

2 Experimental Design

As described in Introduction, our experiment includes one risky lottery, three ambiguity lotteries, two compound lotteries, and two natural-event lotteries as summarized in Table II.

Table II
Summary of Lotteries used in the Experiment

	Lottery	Implementation
R	Risk	A deck of 20 cards with 10 red and 10 black.
A	Full ambiguity	A deck of 20 cards with no information about the composition of red and black.
A1	Interval ambiguity	A deck of 20 cards with the number of red (black) cards between 5 and 15.
A2	Disjoint ambiguity	A deck of 20 cards with the number of red (black) cards either between 0 and 5 or between 15 and 20.
C	Full compound risk	A deck of 20 cards with 21 possible compositions of red and black being equally likely.
C1	p - q compound risk	A deck of 20 cards with $5/8$ ($3/8$) chance of containing 16 (0) red cards and 4 (20) black cards.
T	Natural event (Home)	Trailing digit of home city temperature in a given historical date.
T1	Natural event (Foreign)	Trailing digit of foreign city temperature in a given historical date.

For risk lottery **R**, subjects bet on the color of a card randomly drawn from 20 cards with 10 red and 10 black, and they are explicitly told that the winning probability is 50%. The full ambiguity lottery **A** is the same as that in the Ellsberg paradox, and we include two more partial ambiguity lotteries, i.e., interval ambiguity **A1** and disjoint ambiguity **A2** in which the partial information about the composition of the deck is given to the subjects. The full compound lottery **C** is implemented as follows. One ticket is drawn randomly from a bag containing 21 tickets numbered from 0 to 20, and the number drawn subsequently determines the number of red cards in the deck with the rest black. Similarly, for the p - q compound lottery **C1**, one number is drawn from an envelope containing 8 tickets numbered 1 to 8. If number drawn from is 1 to 5 (6 to 8), the deck will have 16 (0) red cards and 4 (20) black cards. In both compound lotteries, subjects bet on the color of a card drawn from the deck before drawing the ticket, and the winning probability of betting on red (black) can be reduced to 50%. In natural-event lottery **T**, subjects bet on the trailing decimal digit

(odd/even) of the temperature in home city (Singapore) on a randomly selected historical date. Correspondingly, lottery **T1** is based on the temperature of a foreign city (Istanbul).

In our experiment, subjects are allowed to choose the color of the card or odd/even of city temperature to bet, and the psychology of suspicion is less of a concern. For each of the lottery, subjects receive a winning prize SGD60 (about USD43) if guessing correctly and nothing otherwise. To elicit the certainty equivalent (henceforth CE) of each lottery, we use a choice-list design with subjects choosing between a given lottery and a sure amount ranging from SGD15 to SGD35 (see Appendix B for experimental instructions). In general, subjects will have a single switching point, i.e., choosing the lottery when its CE is higher than the sure amount and switch to the sure amount as it increases to the point that is higher than the CE of the lottery. Thus the switch point serves as a proxy for CE of the lottery.

The eight choice-lists are implemented along with the other ten tasks examining risk preference and time preference. Subjects complete the decision making tasks without feedback. To incentivize participation, in addition to a SGD35 show-up fee, we adopt the random incentive mechanism (RIM), paying each subject based on one of her randomly selected decisions in the experiment. The use of RIM to elicit valuations of lotteries has triggered debates from the perspectives of both theory and experiment (see, e.g., Wakker, 2007 for a general discussion, and Baillon, Halevy and Li, 2014 for a discussion of RIM under ambiguity). We adopt this mechanism with price list as it enables us to examine the potential links across different types of uncertainty.

In addition to the decision making tasks, we include a test of cognitive ability using Raven’s Standard Progressive Matrices—one of the most popular measures of analytic intelligence. The test consists of 60 questions that split into five parts of increasing difficulty with 12 questions in each. In each question, subjects are asked to identify a missing element to complete a visual pattern. While we do not impose a time limit, we state that people tend to finish the test in 20–45 minutes. In the literature, higher performance on Raven’s test has been found to be associated with fewer Bayesian updating errors (Charness, Rustichini, and Van de Ven, 2011), more accurate beliefs (Burks et al. 2009), and increased likelihood of choosing strategies that are closer to Nash equilibrium and converge faster to equilibrium play (Gill and Prowse, 2016). The inclusion of Raven’s test enables us to examine a possible relationship between cognitive ability and attitudes towards different types of uncertainty.

We have recruited a cohort of 2066 undergraduate students from National University of Singapore who participated in a study on the biological basis of decision-making (53.0% female; mean age: 21.4). Following the Singapore cohort, an additional cohort of 1181 Han Chinese are recruited from several universities in Beijing (48.4% female; mean age: 21.5). The instructions and procedures are the same for Singapore and Beijing experiments, except for

the following points. First, the prize of the lotteries is changed to RMB240 (about SGD50). Second, for the natural-event lotteries, the home city is Beijing and the foreign city is Tokyo. Last, the written and oral instructions and decision-making tasks are in Chinese instead of English. All subjects provide written informed consent and the experiment was approved by the Institutional Review Board at the National University of Singapore.

3 Theoretical Analysis

This section analyzes how various models of decision making under uncertainty predict individual choice in our experimental setting involving bets arising from: known deck (risk—**R**), unknown deck (ambiguity—**A**), two-stage deck (compound risk—**C**), and city temperature (natural event—**T**). We focus on these four lotteries for expositional simplicity, and the qualitative predictions remain valid when considering the other lotteries. Using **R** as benchmark, we study subjects’ attitudes toward **A**, **C**, and **T**, and the three pairwise correlations of these attitudes, i.e., **A–C**, **A–T**, and **C–T**.

Beginning with subjective expected utility (SEU), we discuss a number of models that can be differentiated in two ways: whether it admits a one-stage or two-stage representation, and whether it incorporates source preference à la Keynes into consideration. We consider representative models under each category. For one-stage models, we examine Gilboa and Schmeidler’s (1989) maxmin expected utility (MEU) and Chew and Sagi’s (2008) model of source preference. For two-stage models, we look at Segal’s (1987) recursive rank-dependent utility without source preference, and two recursive models with source preference across stages: Klibanoff, Marinacci, and Mukerji’s (2005) recursive expected utility and more generally Ergin and Gul’s (2009) second-order probabilistic sophistication.

Denote by S the set of states. Let R and B refer to events red and black, which are subsets of S . We suppress the notation and again use R (B) to denote events odd (even). Let h be an act that maps from S to the set of outcomes. In our experiment, h refers to a bet on R or on B for a given card deck (a city temperature). To facilitate our derivation of the predictions of different models, we impose the following symmetry assumption.

Symmetry: For each lottery, the decision maker is indifferent between betting on R or on B .

Symmetry in terms of an absence of color preference is standard in the literature. While indifference between betting on odd and even seems intuitive, it has been given a preferential foundation via the work of Machina (2004) as follows. For natural-event lottery **T**, the uncertainty associated with the trailing digit of the temperature being either odd or even is ‘almost-objective’ (Machina, 2004) in the sense that for any continuous random variable on the thermometer state space with a smooth density function, the probability of odd or even

approximates 0.5 asymptotically. The intuition is that the event odd (even) in our setting can be viewed as a union of left (right) half of each event in $\{\dots, (-0.1, 0.1], (0.1, 0.3], \dots\}$, which is a partition of the thermometer state space. It follows that the probability of the union event: $\cup_n(\frac{2n-1}{10}, \frac{2n}{10}]$, is approximately 0.5 regardless of the subject's belief about the (smooth) distribution of temperatures given that the partition is fine enough.⁴

Given symmetry, we do not need to differentiate between different bets h (either on R or on B) in our experiment. Consider the benchmark SEU model, the utilities for different lotteries are as follows:

$$U_{SEU}(\mathbf{R}/\mathbf{A}/\mathbf{C}/\mathbf{T}) = \mathbf{E}_p u \circ h,$$

where $p \in \Delta(S)$ denotes the (subjective) prior. SEU, or more generally probabilistic sophistication (Machina and Schmeidler, 1992), entails $p(R) = p(B) = 0.5$ given symmetry. Thus, SEU predicts that four lotteries $\mathbf{R}/\mathbf{A}/\mathbf{C}/\mathbf{T}$ have the same CEs, and hence generate no correlation across attitudes toward ambiguity, compound risk and natural-event uncertainty.

3.1 One-Stage Models

Ellsbergian Perspective. In his 1961 paper, Ellsberg advocates a ‘conservatism’ rule in assigning a positive weight to the worst prior in the unknown bet. Gilboa and Schmeidler’s (1989) MEU model considers the extreme weight of 1, and is widely acknowledged to be the representative model under the Ellsbergian perspective. Under MEU, the utility for ambiguity lottery \mathbf{A} is given below.

$$U_{MEU}(\mathbf{A}) = \min_{p \in \Pi_{\mathbf{A}}} \mathbf{E}_p u \circ h,$$

where $\Pi_{\mathbf{A}}$ is a convex set of priors with $\min_{p \in \Pi_{\mathbf{A}}} p(R) = \min_{p \in \Pi_{\mathbf{A}}} p(B)$ from symmetry. MEU exhibits ambiguity aversion $CE_{\mathbf{R}} \geq CE_{\mathbf{A}}$ as long as $\Pi_{\mathbf{A}}$ is non-singleton.

In relation to compound risk attitude, the MEU model axiomatized in Gilboa and Schmeidler (1989) predicts compound risk neutrality $CE_{\mathbf{R}} = CE_{\mathbf{C}}$, since it adopts the Anscombe-Aumann framework which incorporates RCLA. In contrast, the axiomatization of MEU in a Savagian domain (Casadesus-Masanell, Klibanoff and Ozdenoren, 2000; Alon and Schmeidler, 2014) is silent on how compound lotteries may be evaluated. With regard to natural-event lottery \mathbf{T} , MEU predicts neutrality towards almost-objective uncertainty, i.e., $CE_{\mathbf{R}} \simeq CE_{\mathbf{T}}$.

⁴A general argument for almost objectivity is as follows. First partition the state space into many equal-length intervals, e.g., $\{\dots, (0, \frac{1}{m}], (\frac{1}{m}, \frac{2}{m}], \dots\}$, and then obtain the left half of each interval to form an event as $\cup_n(\frac{n}{m}, \frac{n+0.5}{m}]$. It can be shown that the (subjective) probability of this event converges to 0.5 as m grows large regardless of the prior distribution as long as it is smooth.

In sum, MEU can generate ambiguity non-neutrality, but fails to predict non-neutral attitudes towards either compound risk or almost-objective natural event, and therefore does not generate correlations among different uncertainty attitudes.⁵

Keynesian Perspective. Fox and Tversky (1995) provide evidence in support of Keynes’ source perspective prompting Chew and Sagi (2008, henceforth CS) to axiomatize source preference directly in terms of probabilistic sophistication on smaller families of events, while retaining RCLA. CS views ambiguity lottery \mathbf{A} as an alternative source of uncertainty in addition to \mathbf{R} and \mathbf{T} . Note that we shall be using the term ‘*source of uncertainty*’ only when referring to source preference models in the sequel. The CS model displays flexibility in evaluating \mathbf{R} , \mathbf{A} and \mathbf{T} with source-specific utility functionals:

$$U_{CS}(\mathbf{R}/\mathbf{A}/\mathbf{T}) = \mathbf{U}^s(p, h),$$

where \mathbf{U}^s can admit different (expected or non-expected) utility specifications for different sources of uncertainty even when the prior p is the same. In adopting a source-dependent probability weighting function using rank-dependent utility (Quiggin, 1982), Abdellaoui et al. (2011) offer a more specific form of CS:

$$\mathbf{U}^s(p, h) = \int_S (u \circ h) df^s(\mathbf{P}),$$

where f^s is a source-dependent probability weighting function, and \mathbf{P} the cumulative distribution function derived from p . Notably, CS admits RCLA for compound risk and predicts $CE_{\mathbf{R}} = CE_{\mathbf{C}}$. Given the flexibility in choosing \mathbf{U}^s for different sources of uncertainty, CS can produce correlation between ambiguity attitude and natural-event uncertainty attitude but fails to link compound risk attitude with the attitude towards ambiguity or with attitude towards natural-event uncertainty.

3.2 Two-Stage Models

Two-stage models associate the ambiguity lottery \mathbf{A} with a stage-1 prior $\mu_{\mathbf{A}}$, a distribution on $\Delta(S)$. It follows that for a given act h , $\mu_{\mathbf{A}}$ induces a subjective compound lottery to be evaluated recursively. Here, we discuss Segal’s approach focusing on the subclass of

⁵The CEU model axiomatized in Gilboa (1987) and Schmeidler (1989) generalizes SEU with a capacity function ν , a non-additive extension of a probability measure that maps events into the unit interval and is monotonic in terms of inclusion. In relaxing additivity, CEU is compatible with both ambiguity aversion and ambiguity tolerance without restrictions on the capacities $\nu(R_{\mathbf{A}})$ and $\nu(B_{\mathbf{A}})$. It in fact accommodates MEU given a convex ν .

For compound risk \mathbf{C} , the observations of MEU also apply to CEU models including those adopting the Anscombe-Aumann framework (Schmeidler, 1989) and those adopting a Savagian domain (Gilboa, 1987; Wakker, 1987). In addition, the almost objectivity of events odd/even in \mathbf{T} again entails $CE_{\mathbf{R}} \simeq CE_{\mathbf{T}}$ in the CEU model.

rank-dependent utility followed by recursive models incorporating source preference across stages.

Recursive Rank-Dependent Utility. Segal (1987, 1990) applies a recursive rank-dependent utility specification (RRDU) to evaluate the ambiguity lottery \mathbf{A} as follows:

$$U_{RRDU}(\mathbf{A}) = \int_{\Delta(S)} \int_S (u \circ h) df(\mathbf{P}) d\mathbf{f}(\mathbf{M}_{\mathbf{A}}),$$

where $\mathbf{M}_{\mathbf{A}}(\mathbf{P})$ is the cumulative distribution function derived from $\mu_{\mathbf{A}}(p)$, and $f(u)$ a common probability weighting function (utility function) applied to both stages.

RRDU can exhibit ambiguity non-neutrality through non-reduction behavior for the induced subjective compound lottery. In particular, Segal (1987) shows that a convex f can be compatible with ambiguity aversion $CE_{\mathbf{R}} \geq CE_{\mathbf{A}}$. Given objective uniform stage-1 prior in compound lottery \mathbf{C} , RRDU predicts compound risk aversion $CE_{\mathbf{R}} \geq CE_{\mathbf{C}}$ under the same conditions as that for ambiguity aversion, and hence is compatible with positive correlation between compound risk attitude and ambiguity attitude.

Recursive Source Models. An alternative approach to Segal (1987) is to apply expected utility preference recursively in evaluating the compound lottery. To exhibit non-reduction behavior, this procedure would need to involve two distinct expected utility preferences, which can be deemed as incorporating source preference consideration to differentiate between stage-1 and stage-2 priors. This is accomplished in the recursive expected utility model (REU) of Klibanoff, Marinacci, and Mukerji (2005), Nau (2006), and Seo (2009) as follows:

$$U_{REU}(\mathbf{A}) = \mathbf{E}_{\mu_{\mathbf{A}}} v(\mathbf{E}_p u \circ h),$$

where v and u are stage-1 and stage-2 utility functions respectively, and a concave v is shown to be compatible with ambiguity aversion (Klibanoff, Marinacci and Mukerji, 2005).⁶

REU is further generalized in Ergin and Gul's (2009) second-order probabilistic sophistication representation (SPS) which incorporates possibly distinct NEU preferences across stage-1 and stage-2 priors. Specifically, the utility for ambiguity lottery \mathbf{A} under SPS is as follows:

$$U_{SPS}(\mathbf{A}) = \mathbf{V}(\mu_{\mathbf{A}}, \mathbf{U}(p, h)),$$

where \mathbf{U} is the stage-2 utility functional evaluating act h under a given stage-2 prior p , and \mathbf{V} the stage-1 utility functional that aggregates different stage-2 utilities according to the stage-1 prior $\mu_{\mathbf{A}}$. In SPS, \mathbf{V} and \mathbf{U} both admit probabilistic sophistication, and hence SPS incorporates REU and RRDU as special cases. It follows that the two-stage models considered here, RRDU, REU and SPS, share similar predictions in terms of ambiguity non-neutrality, non-reduction, and correlations between ambiguity attitude and compound

⁶Kreps and Porteus (1978) first propose and axiomatize REU in an intertemporal setting.

risk attitude, regardless of whether source preference is incorporated to distinguish between stage-1 and stage-2 preferences.⁷ When considering almost objective natural-event lottery \mathbf{T} , two-stage models predict natural-event uncertainty neutrality, i.e., $CE_{\mathbf{R}} \simeq CE_{\mathbf{T}}$, and cannot generate correlation between ambiguity/compound risk attitude and natural-event uncertainty attitude.

3.3 Extended Model

Lastly, we consider an extended model combining features of Chew and Sagi (2008) and Ergin and Gul (2009) by distinguishing among within-stage sources of uncertainty (e.g., subjective versus objective) in a two-stage setting.

$$U_{s-SPS}(\mathbf{A}/\mathbf{C}) = \mathbf{V}^s (\mu_{\mathbf{A}/\mathbf{C}}, \mathbf{U}^s(p, h)),$$

where \mathbf{V}^s and \mathbf{U}^s are source-specific stage-1 and stage-2 utility functionals admitting probabilistic sophistication. This source-SPS model inherits the predictions of two-stage models when considering \mathbf{A} and \mathbf{C} , i.e., ambiguity non-neutrality and non-reduction of compound lotteries. In contrast with SPS, source-SPS allows for distinct stage-1 utility functionals, e.g., $\mathbf{V}^{\mathbf{A}}$ for subjective stage-1 prior (in ambiguity) and $\mathbf{V}^{\mathbf{C}}$ for objective stage-1 prior (in compound risk). Moreover, as illustrated below, source-SPS can deliver $CE_{\mathbf{R}} \neq CE_{\mathbf{T}}$ if the stage-2 utility functional $\mathbf{U}^{\mathbf{R}}$ for risk is distinct from $\mathbf{U}^{\mathbf{T}}$ for natural-event uncertainty, even when both sources generate the same even-chance lottery:

$$U_{s-SPS}(\mathbf{R}/\mathbf{T}) = \mathbf{U}^{\mathbf{R}/\mathbf{T}}(p, h).$$

In terms of correlations among different uncertainty attitudes, source-SPS can deliver correlations between attitudes toward ambiguity and compound risk as with SPS. Additionally, source-SPS can deliver correlations between natural-event uncertainty attitude and ambiguity/compound risk attitude if $\mathbf{U}^{\mathbf{R}}$, $\mathbf{U}^{\mathbf{T}}$, $\mathbf{V}^{\mathbf{A}}$ and $\mathbf{V}^{\mathbf{C}}$ are correlated in certain ways. For example, if $\mathbf{U}^{\mathbf{R}}$, $\mathbf{U}^{\mathbf{T}}$ and $\mathbf{V}^{\mathbf{C}}$ all admit expected utility form, and the relative concavity

⁷Note that we focus on the functional forms of recursive models when discussing their predictions on ambiguity attitude, compound risk attitude, as well as the correlation between the two. Formally speaking, the REU model axiomatized in Klibanoff, Mukerji and Marinacci (2005) adopts a Savagean domain and makes no predictions on its behavior in the domain of compound risks. In contrast, the frameworks adopted in Nau (2006) and Ergin and Gul (2009) can in principle admit objective compound risk and accommodate non-RCLA behavior, while both models do not necessarily link such behavior to ambiguity non-neutrality. Seo's (2009) approach of distinguishing between distributions over acts from the state-wise mixture of acts, giving rise to ambiguity and compound risk in a unified domain, is able to link ambiguity non-neutrality with non-RCLA behavior for compound risk. Notably, Halevy and Feltkamp (2005) link ambiguity aversion with aversion to mean-preserving spreads in an environment of bundled risks. Their approach can generate aversion towards both objective and subjective stage-1 priors. See Halevy (2007, p. 515-517) for a related discussion on how various recursive models can(not) differentiate between objective and subjective stage-1 priors.

between the utility indices in \mathbf{U}^R and \mathbf{U}^T is positively correlated with the concavity of the utility index in \mathbf{V}^C , one can have positive correlation between natural-event uncertainty attitude and compound risk attitude.

3.4 Summary

Table III below summarizes the theoretical predictions of different models in terms of attitudes towards the three types of uncertainty (ambiguity/compound lottery/nature-event), as well as pairwise correlations among these attitudes. Except for SEU, all models considered can exhibit ambiguity non-neutrality.

Table III
Theoretical Predictions

Models	Preferences			Correlations		
	Ambiguity	Compound Risk	Nature Event	A – C	A – T	C – T
Multiple-prior	non-Neutral	Neutral/Silent	Neutral	N	N	N
RRDU/REU/SPS	non-Neutral	non-Neutral	Neutral	Y	N	N
CS	non-Neutral	Neutral	non-Neutral	N	Y	N
source-SPS	non-Neutral	non-Neutral	non-Neutral	Y	Y	Y

Note: The table summarizes the qualitative predictions of the models regarding attitudes toward each type of uncertainty, as well as the pairwise correlations among those attitudes. A ‘Y’ for a specific model means it can generate correlations between two corresponding uncertainty attitudes, while a ‘N’ means it either remains silent or is incompatible with correlations between different uncertainty attitudes.

Taking compound risk into consideration, the observed ‘*known*’ link (Halevy, 2007; Abdellaoui, Klibanoff and Placido, 2015; Chew, Miao and Zhong, 2017) between ambiguity attitude and compound risk attitude (RCLA) helps distinguish between two main streams of models in the literature, i.e., one-stage models against two-stage models. In particular, one-stage models including MEU and CS, either permit RCLA or remain silent on how compound lottery should be evaluated, and thus are unable to generate this known link. In contrast, two-stage models, including RRDU, REU and the SPS model, view ambiguity lottery as subjective compound lottery and hence are compatible with the known link.

Considering additionally natural-event uncertainty helps distinguish models via another dimension: whether source preference is incorporated. In particular, both one-stage and two-stage models without source dependence, including MEU and RRDU, fail to generate (significant) difference between an objective half/half lottery based on decks of cards and an ‘almost’ objective half/half lottery based on city temperature. Therefore, they remain silent on the links between natural-event uncertainty attitude and either ambiguity or compound risk attitude.

In contrast, by treating the known deck, unknown deck, and city temperature as alternative sources of even-chance risks, the CS one-stage source model, is able to generate one ‘missing’ link, i.e., between ambiguity and natural-event uncertainty attitudes, but fails to deliver the other two possible links given that it admits RCLA. Notably, REU, or more generally the SPS model, differentiates stage-1 and stage-2 priors as distinct sources of uncertainty, but not the links between natural-event uncertainty attitude and the other two attitudes. This leads us to suggest an extended source-SPS model, which can generate the *known* link through viewing ambiguity as subjective compound lottery, and exhibit the ‘*missing*’ links between natural-event uncertainty attitude and ambiguity (compound risk) attitudes in differentiating among stage-2 sources of uncertainty (deck-based or temperature-based).

4 Results

Summary statistics is presented in Table IV for the eight lotteries (see also Figure A.1 and Figure A.2 in Appendix A). The data is coded as the number of choices in which subjects choose the lottery over the sure amount. A higher number means higher CE with number 7 indicating risk neutrality. We summarize the number of observations, the mean and standard deviation of CE, and the percentage of observations who are neutral, averse and seeking for each lottery.⁸ For the risk lottery **R**, the attitudes of neutrality, aversion and seeking are inferred relative to its expected value of SGD30. For the rest of the lotteries, the attitudes of neutrality, aversion and seeking are inferred by comparing their CEs with $CE_{\mathbf{R}}$.

Table IV
Summary Statistics of Attitudes toward Uncertainty

	Lottery	N	Mean	SD	Neutral	Aversion	Seeking
R	Risk	1943	5.242	2.597	9.4%	75.9%	21.0%
A	Full Ambiguity	2017	2.912	3.043	22.0%	65.1%	12.3%
A1	Interval Ambiguity	1999	3.680	3.000	25.3%	55.8%	18.2%
A2	Disjoint Ambiguity	1996	3.989	3.363	23.3%	53.6%	22.1%
C	Full Compound Risk	2001	3.675	3.250	23.0%	56.3%	19.8%
C1	p - q Compound Risk	2003	4.684	3.380	23.8%	44.2%	30.6%
T	Natural Event (Home)	2010	4.968	2.771	35.7%	35.6%	27.4%
T1	Natural Event (Foreign)	2007	4.665	2.789	35.1%	39.9%	23.9%

We observe that majority of the subjects are ambiguity averse. Among the ambiguity lotteries, CE of full ambiguity lottery: $CE_{\mathbf{A}}$, is significantly smaller than that of interval ambiguity lottery: $CE_{\mathbf{A1}}$ (t-test, $p < 0.001$), as well as that of disjoint ambiguity: $CE_{\mathbf{A2}}$

⁸We observe 2.4% to 6% of choice lists with multiple switch points and exclude these data from the analysis. This results in the difference in number of observations.

(t-test, $p < 0.001$). These replicate the corresponding results in Chew, Miao, and Zhong (2017).

For the compound lotteries, consistent with the findings in previous experimental studies (e.g., Abdellaoui, Klibanoff, and Placido, 2015), we observe that majority of the subjects exhibit compound risk aversion. Between the two compound lotteries, CE of the full compound lottery: CE_C , is significantly smaller than that of the p - q compound lottery: CE_{C1} (t-test, $p < 0.001$). This may reflect the sense that p - q compound lottery is relatively easier to comprehend compared to the full compound lottery.

For natural-event lotteries, while about one third of the subjects exhibit neutrality towards the two temperature lotteries, more subjects are averse to natural-event uncertainty, especially for the foreign temperature lottery. Between these two lotteries, the CE of home temperature lottery CE_T is significantly larger than that of the foreign temperature lottery: CE_{T1} (t-test, $p < 0.001$). This replicates the findings of two previous studies on preference for familiarity (Fox and Tversky, 1995; Abdellaoui et al., 2011).

4.1 New links

We examine the links among attitudes toward ambiguity, compound risk, and natural events. Table V.A reveals a significant association between ambiguity non-neutrality and non-RCLA behavior (Pearson’s chi-squared test, $p < 0.001$). Of 202 subjects who are ambiguity neutral, 124 of them exhibit compound risk neutrality. This is more than six times the expected frequency under the null hypothesis of independence and replicates the studies on the link between ambiguity and compound lottery since Halevy (2007).

Table V.B reveals a significant association between ambiguity neutrality and neutrality towards natural events (Pearson’s chi-squared test, $p < 0.001$). Specifically, of 201 ambiguity neutral subjects, 152 are neutral towards natural events, which is more than 2.5 times the expected frequency under the null hypothesis of independence. This unveils a missing link between ambiguity attitude and attitude towards natural-event uncertainty. Moreover, Table V.C shows a significant association between RCLA behavior and neutrality towards natural-event uncertainty (Pearson’s chi-squared test, $p < 0.001$). Specifically, of 191 subjects exhibiting RCLA, 147 are neutral towards natural-event uncertainty, which is about 3 times the expected frequency under the null hypothesis of independence. This reveals another missing link between attitude towards compound risk and attitude towards natural-event uncertainty.

We further investigate the association among the three uncertainty attitudes by assessing the correlations among the ambiguity premium, the compound risk premium and the natural-

Table V
Linking Attitudes toward Three Types Uncertainty

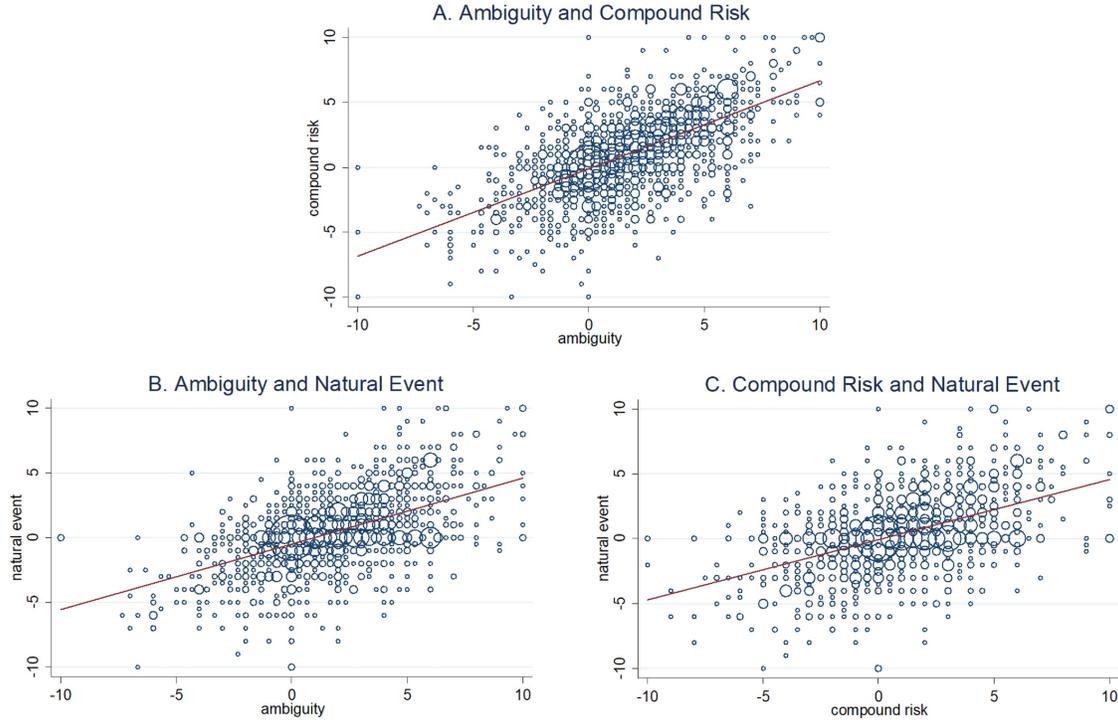
A. Ambiguity and Compound Risk			
Compound Risk			
Ambiguity	Reduction	Non-Reduction	Total
Neutral	124 (20.6)	78 (181.4)	202
Non-Neutral	66 (169.4)	1595 (1492.6)	1,662
Total	190	1,674	1,864
Pearson $\chi^2(1) = 648.59$, $p < 0.001$			
B. Ambiguity and Natural events			
Natural events			
Ambiguity	Neutral	Non-Neutral	Total
Neutral	152 (61.5)	49 (139.5)	201
Non-Neutral	420 (510.5)	1247 (1156.5)	1,667
Total	572	1,296	1,868
Pearson $\chi^2(1) = 214.7$, $p < 0.001$			
C. Natural Events and Compound Risk			
Natural events			
Compound Risk	Reduction	Non-Reduction	Total
Neutral	147 (58.1)	44 (132.9)	191
Non-Neutral	424 (512.9)	1261 (1171.1)	1,685
Total	431	1,463	1,876
Pearson $\chi^2(1) = 217.41$, $p < 0.001$			

Note: The two-way table presents the number of subjects by (non)neutrality towards ambiguity, compound risk and natural-event uncertainty. Each cell indicates the number of subjects with the expected number displayed in parentheses (Pearson’s chi-squared test, $p < 0.001$).

event uncertainty premium. The respective premium is measured by the difference in CEs between the risk lottery and the ambiguity (compound/natural-event) lottery. We find that the correlations of premium across each individual lottery mostly lie between 0.40 and 0.60 (Table A.1 in Appendix A). The average premium in each of the three types of uncertainty and the pair-wise scatter plots are presented in Figure I. We observe significant correlation between average ambiguity premium and average compound risk premium (0.64), between average ambiguity premium and average natural-event uncertainty premium (0.53), and between average compound risk premium and average natural-event uncertainty premium (0.51).

A common concern in preference elicitation is measurement error. Note that the observed correlations are highly significant among the three ambiguity lotteries (0.75, 0.56 and 0.63), between the two compound lotteries (0.48), and between the two natural-event lotteries

Figure I
 Pair-wise Scatter Plots of Average Premium across Three Types of Uncertainty



(0.86), This suggests common underpinnings within each type of uncertainty in conjunction with measurement errors. In subsequent analysis, we make use of the method of Obviously Related Instrumental Variables (ORIV) developed in Gillen, Snowberg and Yariv (2018) to correct for measurement error given that we have multiple elicitations of closely related attitudes.⁹ Specifically, we have three ambiguity lotteries, two compound lotteries, and two natural-event lotteries that can serve as multiple elicitations. As reported in Table I in the Introduction, the correlations are substantially improved adopting ORIV: correlation between ambiguity premium and compound risk premium increases to 0.82, correlation between ambiguity premium and natural-event uncertainty premium increases to 0.57, and correlation between compound risk premium and natural-event uncertainty premium increases to 0.66.¹⁰ This is in line with the observations in Gillen, Snowberg and Yariv (2018) which reports an increase in correlation between ambiguity premium and compound risk premium from 0.44 to 0.85 using ORIV. Overall, these analyses further strengthen the links among these three

⁹ORIV adopts an errors-in-variable instrumental variable approach to analyze the data when there are multiple elicitations of the behavioral proxies.

¹⁰In addition, we also use principle component analysis within each source of uncertainty and use the first component to conduct the correlation analysis. The correlation is 0.64 between ambiguity premium and compound risk premium, 0.54 between ambiguity premium and natural-event uncertainty premium, and 0.51 between compound risk premium and natural-event uncertainty premium.

uncertainty attitudes.

Finally, we examine the behavior at the individual level (Table VI). A subject is classified as being neutral towards a particular type of uncertainty if the CEs for lotteries in that type of uncertainty are the same as the CE for the risk lottery. We find that 66.0% of the subjects are non-neutral towards all three types of uncertainty, while 5.7% of the subjects are neutral towards all three. For the rest of subjects, 20.4% are non-neutral towards ambiguity and compound risk, and neutral towards natural-event uncertainty. These three groups of subjects account for 92% of the subjects, with the rest of 8% exhibiting other behavioral patterns.

Table VI
Individual Type Analysis

Ambiguity	Compound Risk	Natural Event	Percent
Non-Neutral	Non-Neutral	Non-Neutral	66.0%
Non-Neutral	Non-Neutral	Neutral	20.4%
Neutral	Neutral	Neutral	5.7%
Non-Neutral	Neutral	Non-Neutral	1.4%
Neutral	Non-Neutral	Non-Neutral	1.8%
Non-Neutral	Neutral	Neutral	2.2%
Neutral	Non-Neutral	Neutral	2.5%
Neutral	Neutral	Non-Neutral	0.9%

Note. The table presents the percentage of individual type classified by attitudes toward three types of uncertainty. Neutrality towards certain type of uncertainty corresponds to the same CE between the risky lottery and each of the lotteries in that type of uncertainty.

4.2 Cognitive ability

In this subsection, we explore the role of cognitive ability measured by the scores in Raven’s test (mean = 56.02, median = 57; SD = 4.60) by comparing subjects with high scores and those with low scores (1059 subjects with scores larger than 57; 1022 subjects with scores smaller than or equal to 57). We report the statistics in Tables A.2 to A.4 in Appendix A, and summarize the results here. Overall, our results are robust in the following sense. First, regardless of the levels of cognitive ability, correlation remains high across three types of uncertainty and it is higher between attitudes towards ambiguity and compound risk. Second, a strong majority of the subjects remain non-neutral towards the three types of uncertainty, followed by those subjects who are non-neutral towards ambiguity and compound risk, but neutral towards natural-event uncertainty, with neutrality towards all three types coming in at a distant third.

We further examine the role of cognitive ability, we first examine how cognitive ability is related to attitude towards ambiguity, compound risk and natural-event uncertainty, and we find one out of seven lotteries exhibiting significant difference. More specifically, subjects with high scores tend to have higher premium for full ambiguity, compared with those with low scores ($p < 0.038$). We then examine the pairwise correlations across the three types of uncertainty, and find that above-median subjects are linked to higher correlation between ambiguity attitude and compound risk attitude, and lower correlation between ambiguity attitude and natural-event uncertainty attitude, as well as between compound risk attitude and natural-event uncertainty attitude. Lastly, in individual type analysis, among subjects exhibiting non-neutrality towards all three types of uncertainty, the percentage of 63.4% for above-median subjects is significantly lower than 67.3% for below-median subjects ($p < 0.063$). For subjects exhibiting neutrality towards all three types of uncertainty, the percentage is 6.6% for above-median subjects and 4.6% for below-median subjects ($p < 0.048$). In summary, close links are observed across types of uncertainty with respect to cognitive ability. Moreover, subjects with high scores are less (more) likely to be non-neutral (neutral) towards all three types of uncertainty.

4.3 Replicability

In this subsection, we examine the replicability of our findings using the Beijing sample. We display the results of our analysis in Tables A.5 to A.7 in Appendix A and summarize them here. Overall, our results are replicated in Beijing sample in terms of the high correlations among three types of uncertainty and the similarity in the percentages of individual types. To explore the difference between Beijing and Singapore samples, we first examine the attitudes towards ambiguity, compound risk, and natural-event uncertainty. Despite statistical significance, the difference between Singapore and Beijing samples in the percentages of being aversion is generally less than 3%. One exception is with the p - q compound lottery, in which we find that the percentage of being aversion is 44.2% among Singapore subjects compared to 31.5% among Beijing subjects. Examining the pairwise correlations across the different types of uncertainty, we find that the correlation is 0.72 between ambiguity attitude and compound risk attitude, 0.59 between ambiguity attitude and natural-event attitude, and 0.63 between compound risk attitude and natural-event attitude. Lastly, in individual type analysis, we find that 68.5% of Beijing subjects exhibit non-neutrality towards all three types of uncertainty, and 16.5% exhibit non-neutrality towards ambiguity and compound risk, and neutrality towards natural-event uncertainty, and 4.0% are neutral towards all three types of uncertainty. In addition, the observed patterns with regard to cognitive ability

are also replicated in Beijing sample (see Tables A.8 to A.10 in Appendix A). Overall, the main choice behavior of Beijing subjects remain similar to what is observed for Singapore subjects. This said, Beijing subjects are less averse to p - q compound lottery, exhibit lower correlation between ambiguity attitude and compound risk attitude, and less likely to exhibit non-neutrality towards ambiguity and compound risk and neutrality towards natural events.

5 Discussion

Almost Objective Uncertainty. Building on the works of Tversky and his colleagues (Heath and Tversky, 1991; Fox and Tversky, 1995), a number of recent studies examine attitude towards natural-event uncertainty. Adopting the definition of event exchangeability in Chew and Sagi (2008), Abdellaoui et al. (2011) use a bisection procedure for subjects to partition the state space, e.g., the range of home (foreign) city temperature, into disjoint intervals with equal likelihoods. They observe a substantial proportion of subjects exhibiting preference for betting on the temperature in their home city rather than a foreign city. Relating to the opening quote of Keynes, subjects in their study may possess different degrees of confidence or affinity with the source—home versus foreign city—of the underlying temperature uncertainty, despite their likelihoods being assessed as equal. In this design, models with multiple priors can still account for the observed preference for the familiar with different (distributions on the) sets of priors based on the degree of confidence in the assessed probability.

In our experiment, the uncertainty associated with the trailing digit being odd or even is almost-objective, and it seems unlikely for subjects to hold different degrees of confidence in assessing the likelihoods in terms of home or foreign city temperature. Yet, we observe a preference for the risk lottery to these two almost-objective temperature lotteries, as well as a significant but weaker preference to bet on home temperature rather than on foreign temperature. Relatedly, Machina (2014) proposes a Thermometer Problem, which considers bets on the temperature in Timbuktu measured by a hypothetical thermometer that can report any value in the continuum. When allowed to divide the state space into an extremely large number of equal-length intervals, Machina (2014) shows that the Allais-type behavior such as common-ratio and common-consequence in the objective risk domain can also occur for almost objective uncertainty arising from bets on the temperature. These observations challenge ambiguity models adopting multiple priors whose preferences over almost-objective acts approaches asymptotically to objective expected utility. By contrast, the CS source preference approach is compatible with the wide range of phenomena mentioned here through modeling decision makers having preference over distinct sources of uncertainty, e.g., temperature in different cities regardless of familiarity or almost objectivity nature.

Linking Domains of Uncertainty. Despite its flexibility in viewing each type of uncertainty as distinct (one-stage) sources, the source preference approach does not offer a plausible account for the stronger correlation in attitudes between ambiguity and compound risk (0.82) than that between ambiguity and natural events (0.56) as well as between compound risk and natural events (0.66). Notably, the observed link between attitudes toward full ambiguity and full compound risk may admit an alternative interpretation that the compound lottery may be viewed as ambiguity due to the similarity in presentation or its complexity. Here we observe a significant correlation in attitudes between full ambiguity and p - q compound risk (0.42), which is not substantially different from that between full ambiguity and full compound risk (0.55). As it is unlikely for the subjects to view the p - q compound lottery as ambiguity, our results support the view that subjects view ambiguity as compound lottery. In this regard, our findings corroborate Halevy’s (2007) observed close link between ambiguity attitude and compound risk attitude and lend further support to the two-stage modelling of decision making under ambiguity.¹¹

A number of recent studies can be interpreted as supporting links across types of uncertainty. Armantier and Treich (2015) consider a ‘complex risk’ treatment, in which the outcome of the bet depends on the color of two balls simultaneously drawn from two risky urns. They observe a tight association between attitude towards this complex risk and attitude towards ambiguity, and suggest that their finding may hint on the link between ambiguity attitude and complexity attitude. Epstein and Halevy (2017) examine bets depending on the color of two balls drawn from two ambiguous urns and find that attitude towards ambiguous correlation between two unknown urns is correlated with attitude towards the standard Ellsbergian ambiguity. Li et al. (2017) examine uncertainty arising from betting on the home district of an Indian rural child. Given that subjects have no prior knowledge on different India districts, they observe a significant correlation between attitudes toward betting on the districts and betting on an unknown urn.

Cognitive and consequentialist considerations. Implicit in the formulation of the expected utility hypothesis is an exclusive focus on what may be considered as consequences in an uncertain alternative, chiefly the outcomes and their associated likelihoods. This consequentialist focus is also evident in the development of non-expected utility models, culminating in the definition of probabilistic sophistication axiomatized in Machina and Schmeidler (1992). It has been suggested that departures from the prescriptions of expected utility, such as Allais and Ellsbergian behavior, exemplifies non-consequentialist behavior and may have its roots

¹¹This link has been examined and largely replicated in a number of recent studies (Dean and Ortoleva, 2016; Abdellaoui, Klibanoff, and Placido, 2015; Gillen, Snowberg, and Yariv, 2018; Chew, Miao, and Zhong, 2017).

in limitations in subjects’ cognitive ability or attentiveness (see, e.g., Morgenstern, 1979). To address this cognitive limitation hypothesis in conjunction with ambiguity attitude elicited using the two-urn problem, subjects in Chew, Ratchford, and Sagi (2017) are first screened for comprehension and attentiveness before being presented with a matrix version of the two-urn problem intended to minimize the difference in complexity between the known and the unknown bets. They find that low-comprehension subjects are seemingly ambiguity neutral in choosing randomly while seven-tenths of high-comprehension subjects continue to favor the known bet. This rate of ambiguity aversion is in line with what is reported in the literature.¹² As discussed earlier, 57 out of 60 is the median score of our subjects in Ravens IQ test following the choice tasks. This suggests that the bulk of our subjects are high-comprehension and attentive. Yet, we continue to observe systematic departures from consequentialist behavior in attitudes toward the three types of uncertainty, including the strong pairwise correlations, among our subjects as well as for the subsample of above-median subjects.¹³ Taken together, our findings reveal that decision making under uncertainty can depend on how choice is perceived even when there is sufficient attention and comprehension.

6 Conclusion

The discovery of the unknown urn by Ellsberg and Keynes has spawned a voluminous literature on decision making under uncertainty, encompassing ambiguity, compound risk, and natural-event uncertainty, giving rise to distinct strands of papers. The present paper examines experimentally attitudes toward these distinct types of uncertainty and find that subjects exhibit considerable heterogeneity among them. There is a tendency to be non-neutral towards each type of uncertainty and each pair of these attitudes tend to be highly correlated. In this regard, attitude towards one type of uncertainty may be informative in predicting attitude over another type of uncertainty. Our findings contribute to an emerging sense of the role of perception in decision making under uncertainty. Whether a bet on the unknown urn is perceived as being probabilistic or having multiple priors, and whether multiple priors may in turn be perceived as a compound lottery can materially impact how

¹²It is instructive to revisit Raiffa’s (1961) reflection on his ‘error’ in favoring the known bet when responding to Ellsberg’s request to provide his choice in the two-urn problem “without any pencil pushing”. He realized afterwards that he could fashion from the unknown bet a two-stage lottery with overall winning probability of 1/2 by betting red if a coin flip turns out head and bet black otherwise. Consistent with RCLA, Raiffa would be indifferent between the two-stage bet and the known bet. Raiffa’s reflection points to an innate tendency to avoid ambiguity when he is not particularly attentive while there is little doubt about his ability to behave in a consequentialist manner with sufficient attention.

¹³In this regard, the finding in Abdellaoui, Klibanoff, and Placido (2011) of the ambiguity-compound risk link being stronger for non-engineers than engineers may be due to their greater engagement and attentiveness.

it is evaluated. Taken together, our findings point to the need for theoretical modelling of decision making under uncertainty to incorporate how decision makers may perceive an act in terms of its underlying sources of uncertainty as well as whether and how attitudes toward such uncertainties are linked.

REFERENCES

- [1] Abdellaoui, Mohammed, Aurélien Baillon, Laetitia Placido, and Peter P. Wakker, “The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation.” *American Economic Review*, 101(2011), 695–723.
- [2] Abdellaoui, Mohammed, Peter Klibanoff, and Laetitia Placido, “Experiments on Compound Risk in Relation to Simple Risk and to Ambiguity.” *Management Science*, 61(2015), 1306–1322.
- [3] Alon, Shiri, and David Schmeidler, “Purely Subjective Maxmin Expected Utility.” *Journal of Economic Theory*, 152(2014), 382–412.
- [4] Anscombe, Francis J., and Robert J. Aumann, “A Definition of Subjective Probability.” *Annals of mathematical statistics*, 34(1963), 199–205.
- [5] Armantier, Olivier, and Nicolas Treich, “The rich domain of risk.” *Management Science*, 62(2015), 1954-1969.
- [6] Baillon, Aurélien, Yoram Halevy, and Chen Li, “Experimental Elicitation of Ambiguity Attitude using the Random Incentive System.” *Working Paper*, 2014.
- [7] Becker, Selwyn W., and Fred O. Brownson, “What Price Ambiguity? Or the Role of Ambiguity in Decision-Making.” *Journal of Political Economy*, 72(1964), 62–73.
- [8] Burks, Stephen V., Jeffrey P. Carpenter, Lorenz Goette, and Aldo Rustichini. “Cognitive skills affect economic preferences, strategic behavior, and job attachment.” *Proceedings of the National Academy of Sciences*, 106(2009), 7745-7750.
- [9] Camerer, Colin, and Martin Weber, “Recent developments in modeling preferences: Uncertainty and ambiguity.” *Journal of risk and uncertainty*, 5(1992): 325-370.
- [10] Casadesus-Masanell, Ramon, Peter Klibanoff, and Emre Ozdenoren, “Maxmin Expected Utility over Savage Acts with a set of Priors.” *Journal of Economic Theory*, 92(2000), 35–65.
- [11] Charness, Gary, Aldo Rustichini, and Jeroen Van de Ven, “Self-confidence and strategic deterrence.” *Working Paper*, 2011.
- [12] Chew, Soo Hong, Richard P. Ebstein, and Songfa Zhong, “Ambiguity aversion and familiarity bias: Evidence from behavioral and gene association studies.” *Journal of Risk and Uncertainty*, 44(2012), 1-18.
- [13] Chew, Soo Hong, Bin Miao, and Songfa Zhong, “Partial Ambiguity.” *Econometrica*, 85(2017), 1239-1260.

- [14] Chew, Soo Hong, Mark Ratchford, and Jacob S. Sagi, “You Need to Recognize Ambiguity to Avoid It.” *The Economic Journal*, 2017.
- [15] Chew, Soo Hong, and Jacob S. Sagi, “Small Worlds: Modeling Preference over Sources of Uncertainty.” *Journal of Economic Theory*, 139(2008), 1–24.
- [16] Dean, Mark, and Pietro Ortoleva, “Is it All Connected? A Testing Ground for Unified Theories of Behavioral Economics Phenomena.” *Working Paper*, (2016).
- [17] Dimmock, Stephen G., Roy Kouwenberg, Olivia S. Mitchell, and Kim Peijnenburg, “Ambiguity aversion and household portfolio choice puzzles: Empirical evidence.” *Journal of Financial Economics*, 119(2016), 559–577.
- [18] Dimmock, Stephen G., Roy Kouwenberg, and Peter P. Wakker, “Ambiguity attitudes in a large representative sample.” *Management Science* 62(2015), 1363–1380.
- [19] Ellsberg, Daniel, “Risk, Ambiguity, and the Savage Axioms.” *Quarterly Journal of Economics*, 75(1961), 643–669.
- [20] Epstein, Larry G., and Yoram Halevy, “Ambiguous Correlation.” *Review of Economic Studies*, forthcoming, 2017.
- [21] Ergin, Haluk, and Faruk Gul, “A Theory of Subjective Compound Lotteries.” *Journal of Economic Theory*, 144(2009), 899–929.
- [22] Fox, Craig R., and Amos Tversky, “Ambiguity Aversion and Comparative Ignorance,” *The Quarterly Journal of Economics*, 110(1995), 585–603.
- [23] Gajdos, Thibault, Takashi Hayashi, J-M. Tallon, and J-C. Vergnaud, “Attitude toward Imprecise Information.” *Journal of Economic Theory*, 140(2008), 23–56.
- [24] Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci, “Differentiating Ambiguity and Ambiguity Attitude.” *Journal of Economic Theory*, 118(2004), 133–173.
- [25] Gilboa, Itzhak, *Making better decisions: Decision theory in practice*. John Wiley & Sons, 2010.
- [26] Gilboa, Itzhak, “Expected Utility with Purely Subjective Nonadditive Probabilities.” *Journal of Mathematical Economics*, 16(1987), 65–88.
- [27] Gilboa, Itzhak., and David Schmeidler, “Maxmin expected Utility with Non-unique Prior.” *Journal of Mathematical Economics*, 18(1989), 141–153.
- [28] Gill, David, and Victoria Prowse, “Cognitive ability, character skills, and learning to play equilibrium: A level-k analysis.” *Journal of Political Economy*, 124(2016), 1619–1676.
- [29] Gillen, Ben, Erik Snowberg, and Leeat Yariv, “Experimenting with Measurement Error: Techniques with Applications to the Caltech Cohort Study.” *Journal of Political Economy*, forthcoming, 2018.
- [30] Halevy, Yoram, “Ellsberg Revisited: An Experimental Study.” *Econometrica*, 75(2007), 503–536.
- [31] Halevy, Yoram, and Vincent Feltkamp, “A Bayesian Approach to Uncertainty Aversion.” *The Review of Economic Studies*, 72(2005), 449–466.

- [32] Heath, Chip, and Amos Tversky, “Preference and Belief: Ambiguity and Competence in Choice under Uncertainty.” *Journal of Risk and Uncertainty*, 4(1991), 5-28.
- [33] Keynes, John Maynard, *A treatise on probability*. London: MacMillan, 1921.
- [34] Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji, “A Smooth Model of Decision Making under Ambiguity,” *Econometrica*, 73(2005), 1849–1892.
- [35] Kreps, David M., and Evan L. Porteus, “Temporal Resolution of Uncertainty and Dynamic Choice Theory,” *Econometrica*, 46(1978), 185–200.
- [36] Li, Zhihua, Julia Muller, Peter P. Wakker, and Tong Wang, “The Rich Domain of Uncertainty Explored,” *Management Science*, forthcoming, (2017).
- [37] Loewenstein, George F., Elke U. Weber, Christopher K. Hsee, and Ned Welch, “Risk as feelings,” *Psychological Bulletin*, 127(2001), 267-286.
- [38] Machina, Mark J., “Almost-Objective Uncertainty.” *Economic Theory*, 24(2004), 1–54.
- [39] Machina, Mark J., “Ambiguity aversion with three or more outcomes.” *American Economic Review*, 104(2014), 3814-40.
- [40] Machina, Mark J., and David Schmeidler, “A more robust definition of subjective probability.” *Econometrica*, 60(1992), 745–780.
- [41] Morgenstern, Oskar, “Some reflections on utility.” *Expected utility hypotheses and the allais paradox* (pp. 175-183). Springer, Dordrecht, 1979.
- [42] Nau, Robert F., “Uncertainty Aversion with Second-Order Utilities and Probabilities.” *Management Science*, 52(2006), 136–145.
- [43] Pratt, John W., “Risk Aversion in the Small and in the Large.” *Econometrica*, 32(1964), 122-136.
- [44] Quiggin, John, “A Theory of Anticipated Utility.” *Journal of Economic Behavior and Organization*, 3(1982), 323–343.
- [45] Raiffa, Howard, “Risk, ambiguity, and the Savage axioms: comment.” *The Quarterly Journal of Economics*, 75(1961), 690-694.
- [46] Schmeidler, David, “Subjective Probability and Expected Utility without Additivity.” *Econometrica*, 57(1989), 571–587.
- [47] Segal, Uzi, “The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach.” *International Economic Review*, 28(1987), 175–202.
- [48] ———, “Two-Stage Lotteries without the Reduction Axiom.” *Econometrica*, 58(1990), 349–377.
- [49] Seo, Kyoungwon, “Ambiguity and Second-Order Belief.” *Econometrica*, 77(2009), 1575–1605.
- [50] Smith, Vernon L., “Measuring nonmonetary utilities in uncertain choices: The Ellsberg urn.” *The Quarterly Journal of Economics*, 83(1969), 324-329.
- [51] Trautmann, Stefan T., and Gijs Van De Kuilen, “Ambiguity attitudes.” *The Wiley Blackwell handbook of judgment and decision making*, 1(2015), 89-116.

- [52] Wakker, Peter P., “Nonadditive Probabilities and Derived Strengths of Preferences.” *unpublished manuscript*, 1987.
- [53] Wakker, Peter P., “Message to Referees Who Want to Embark on yet Another Discussion of the Random-lottery Incentive System For Individual Choice.” <http://people.few.eur.nl/wakker/miscella/debates/randomlinc.htm>, 2007.
- [54] Wakker, Peter P., *Prospect theory: For risk and ambiguity*. Cambridge university press, 2010.
- [55] Warnick, James C. Engle, Javier Escobal, and Sonia C. Laszlo, “Ambiguity aversion and portfolio choice in small-scale Peruvian farming.” *The BE Journal of Economic Analysis and Policy*, 11(2011), 1.

Appendix A: Supplementary Tables and Figures

Table A.1

Spearman correlations across the premium of each lottery

	A	A1	A2	C	C1	T
A1	<i>0.745</i>					
A2	<i>0.556</i>	<i>0.631</i>				
C	0.549	0.570	0.500			
C1	0.424	0.447	0.418	<i>0.484</i>		
T	0.445	0.469	0.471	0.463	0.406	
T1	0.440	0.452	0.441	0.450	0.382	<i>0.864</i>

Note. The premium is computed as the difference between the CE of the risky lottery and that of the corresponding lottery.

Table A.2
Attitudes toward uncertainty by cognitive ability

Lottery	Low Scores			High Scores			p-value
	Neutral	Aversion	Seeking	Neutral	Aversion	Seeking	
A Full Ambiguity	21.4%	63.8%	14.7%	22.6%	66.0%	11.4%	0.033
A1 Interval Ambiguity	24.3%	54.0%	21.7%	26.0%	57.3%	16.7%	0.074
A2 Disjoint Ambiguity	24.8%	50.8%	24.4%	21.8%	56.2%	22.1%	0.278
C Full Compound Risk	24.1%	55.5%	20.4%	22.2%	57.0%	20.9%	0.292
C1 p - q Compound Risk	23.1%	42.2%	34.7%	24.4%	46.2%	29.4%	0.056
T Natural Event (Home)	33.0%	37.3%	29.7%	38.5%	33.7%	27.8%	0.541
T1 Natural Event (Foreign)	32.2%	42.3%	25.5%	38.2%	37.5%	24.3%	0.574

Note. The table presents the percentage of different attitudes toward each lottery by low cognitive ability (scores ≤ 57 , $N = 1022$) and high cognitive ability (scores > 57 , $N = 1059$). The last column reports the p-value using regression analysis with premium as dependent variable and score as independent variable.

Table A.3
Correlations across three types of of uncertainty by cognitive ability

	Low Scores		High Scores	
	Ambiguity	Compound Risk	Ambiguity	Compound Risk
Compound Risk	0.803 (0.014)		0.842 (0.018)	
Natural Event	0.589 (0.014)	0.714 (0.017)	0.541 (0.012)	0.604 (0.015)

Note: The table presents the correlations for attitudes toward the three types of uncertainty by low cognitive ability (scores ≤ 57 , $N = 1022$) and high cognitive ability (scores > 57 , $N = 1059$).

Table A.4
Individual types by cognitive ability

Ambiguity	Compound Risk	Natural Event	Low Scores	High Scores	p-value
Non-Neutral	Non-Neutral	Non-Neutral	67.3%	63.4%	0.062
Non-Neutral	Non-Neutral	Neutral	19.1%	21.5%	0.173
Neutral	Neutral	Neutral	4.6%	6.6%	0.048
Non-Neutral	Neutral	Non-Neutral	2.3%	0.5%	0.001
Neutral	Non-Neutral	Non-Neutral	2.3%	1.4%	0.127
Non-Neutral	Neutral	Neutral	2.0%	2.4%	0.534
Neutral	Non-Neutral	Neutral	1.7%	3.3%	0.020
Neutral	Neutral	Non-Neutral	0.8%	0.9%	0.804

Note. The table presents the percentage of individual type classified by attitudes toward three types of uncertainty, by low cognitive ability (scores ≤ 57 , $N = 1022$) and high cognitive ability (scores > 57 , $N = 1059$).

Table A.5
Attitudes toward uncertainty: Singapore and Beijing subjects

Lottery	Singapore			Beijing			p-value
	Neutral	Aversion	Seeking	Neutral	Aversion	Seeking	
A Full Ambiguity	22.0%	65.1%	13.0%	23.9%	63.2%	12.8%	0.001
A1 Interval Ambiguity	25.3%	55.8%	18.9%	25.6%	53.2%	21.1%	0.001
A2 Disjoint Ambiguity	23.3%	53.6%	23.1%	21.7%	52.0%	26.3%	0.001
C Full Compound Risk	23.0%	56.3%	20.7%	25.1%	55.6%	19.3%	0.003
C1 p - q Compound Risk	23.8%	44.2%	32.0%	23.5%	31.5%	45.0%	0.001
T Natural Event (Home)	35.7%	35.6%	28.7%	33.8%	34.2%	32.0%	0.207
T1 Natural Event (Foreign)	35.1%	39.9%	24.9%	33.4%	36.9%	29.7%	0.013

Note. The table presents the percentage of different attitudes toward each lottery, comparing Singapore and Beijing subjects.

Table A.6
Correlations across three types of uncertainty: Singapore and Beijing subjects

	Singapore		Beijing	
	Ambiguity	Compound Risk	Ambiguity	Compound Risk
Compound Risk	0.823 (0.014)		0.723 (0.018)	
Natural Event	0.563 (0.009)	0.657 (0.011)	0.593 (0.014)	0.620 (0.016)

Note. The table presents the correlations across attitudes toward three types of uncertainty, comparing Singapore and Beijing subjects.

Table A.7
Individual types: Singapore and Beijing subjects

Ambiguity	Compound Risk	Natural Event	Singapore	Beijing	p-value
Non-Neutral	Non-Neutral	Non-Neutral	66.0%	68.5%	0.144
Non-Neutral	Non-Neutral	Neutral	20.4%	16.5%	0.006
Neutral	Neutral	Neutral	5.7%	4.0%	0.033
Non-Neutral	Neutral	Non-Neutral	1.4%	2.8%	0.005
Neutral	Non-Neutral	Non-Neutral	1.8%	2.0%	0.685
Non-Neutral	Neutral	Neutral	2.2%	3.2%	0.082
Neutral	Non-Neutral	Neutral	2.5%	2.0%	0.361
Neutral	Neutral	Non-Neutral	0.9%	1.0%	0.775

Note. The table presents the percentage of individual type classified by attitudes toward three types of uncertainty, comparing Singapore and Beijing subjects.

Table A.8
Attitudes toward uncertainty by cognitive ability (Beijing subjects)

Lottery	Low Scores			High Scores			p-value
	Neutral	Aversion	Seeking	Neutral	Aversion	Seeking	
A Full Ambiguity	25.2%	63.3%	11.4%	23.3%	62.8%	13.8%	0.698
A1 Interval Ambiguity	25.1%	54.7%	20.2%	26.1%	52.1%	21.8%	0.186
A2 Disjoint Ambiguity	20.7%	51.6%	27.7%	22.4%	52.2%	25.4%	0.122
C Full Compound Risk	23.1%	59.4%	17.5%	26.5%	52.7%	20.8%	0.041
C1 p - q Compound Risk	20.5%	34.2%	45.3%	25.3%	29.7%	45.0%	0.394
T Natural Event (Home)	31.9%	36.4%	31.7%	35.3%	32.8%	31.9%	0.031
T1 Natural Event (Foreign)	31.8%	38.1%	30.1%	34.6%	36.2%	29.2%	0.065

Note. The table presents the percentage of attitude toward each lottery for Beijing subjects, by low cognitive ability (scores ≤ 56 , $N = 526$) and high cognitive ability (scores > 56 , $N = 663$). The last column reports the p-value using regression analysis with premium as dependent variable and score as independent variable.

Table A.9
Correlations across three types of uncertainty by cognitive ability (Beijing subjects)

	Low Scores		High Scores	
	Ambiguity	Compound Risk	Ambiguity	Compound Risk
Compound Risk	0.710 (0.029)		0.733 (0.023)	
Natural Event	0.622 (0.024)	0.650 (0.026)	0.586 (0.017)	0.611 (0.021)

Note: The table presents the correlations for attitudes towards the three types of uncertainty for Beijing subjects, by low cognitive ability (scores ≤ 56 , $N = 526$) and high cognitive ability (scores > 56 , $N = 663$).

Table A.10
Individual types by cognitive ability (Beijing subjects)

Ambiguity	Compound Risk	Natural Event	Low Scores	High Scores	p-value
Non-Neutral	Non-Neutral	Non-Neutral	67.9%	61.2%	0.039
Non-Neutral	Non-Neutral	Neutral	19.6%	22.1%	0.323
Neutral	Neutral	Neutral	2.5%	5.0%	0.038
Non-Neutral	Neutral	Non-Neutral	2.0%	3.3%	0.219
Neutral	Non-Neutral	Non-Neutral	1.5%	2.2%	0.830
Non-Neutral	Neutral	Neutral	3.4%	3.1%	0.793
Neutral	Non-Neutral	Neutral	2.0%	2.2%	0.428
Neutral	Neutral	Non-Neutral	1.2%	0.9%	0.644

Note. The table presents the percentage of individual type classified by attitudes toward three types of uncertainty for Beijing subjects, by low cognitive ability (scores ≤ 56 , $N = 526$) and high cognitive ability (scores > 56 , $N = 663$).

Figure A.1
 Distribution of switching points for each lottery

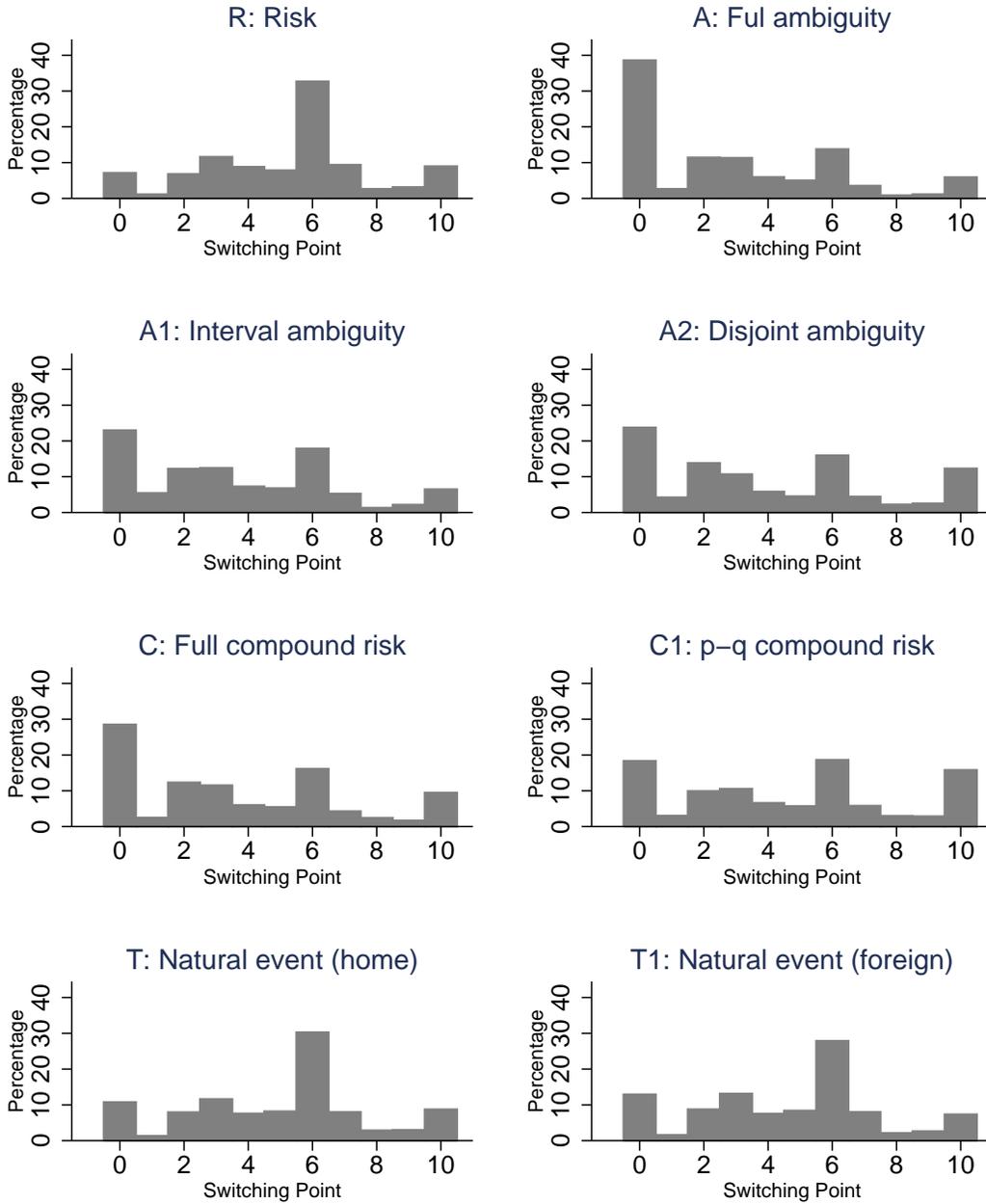
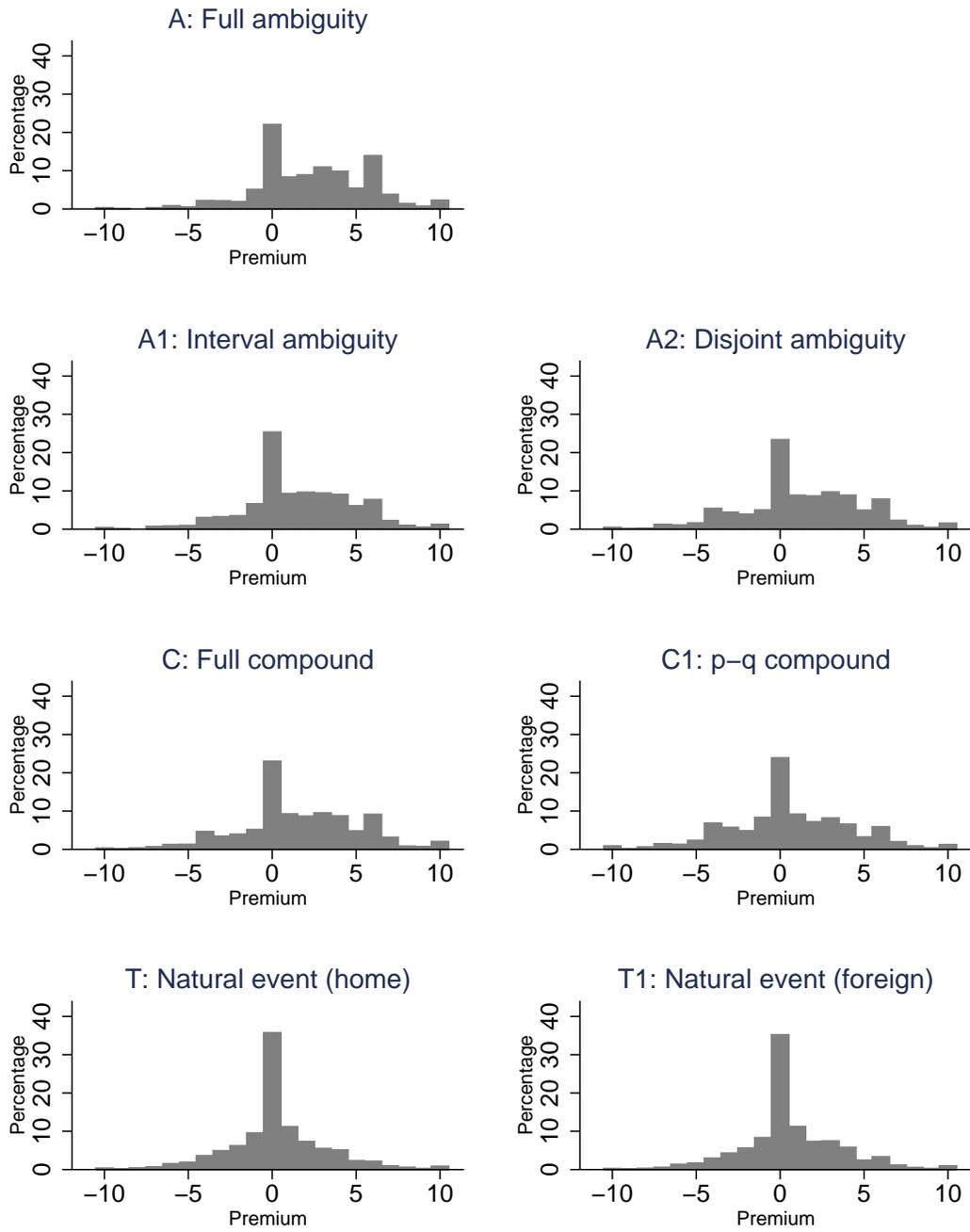


Figure A.2
Distribution of uncertainty premium for each lottery



Appendix B: Experimental Instructions¹

GENERAL INSTRUCTIONS

Welcome to our study on decision making. The descriptions of the study contained in this experimental instrument will be implemented fully and faithfully.

You will go through 3 stages in this study. **Stage 1** (i.e., today) is a 2-hour study consisting of **3 sets** of tasks. The **first set** comprises **19 individual decision making tasks**. The **second set** is made up of **11 decision making tasks** involving other participants in this room. The **third set** consists of a **questionnaire**.

Stage 2 is an online study involving both decision making tasks and questionnaires. After completing stage 2, you can sign up for a 30-minute time slot **during 8 – 19 November** to participate in **Stage 3**. You will receive your **overall compensation** during Stage 3.

Each participant will **receive on average \$80** for participation in the study. Your **actual compensation** includes a **\$35 show up fee** in addition to **earnings** and **losses** based on how you and others make decisions.

All information provided will be kept CONFIDENTIAL. Information in the study including your personal information as well as your decisions will be used for research purposes only.

1. The set of decision making tasks and the instructions for each task are the **same** for all participants. Please refrain from discussing with anyone any aspect of the specific tasks during or after the study.
2. This experimental instrument is printed **double-sided**. Please go through the instructions **carefully** to understand the tasks fully and make informed decisions.
3. At any time, if you have questions, please **raise your hand**.
4. Please **do not communicate** with other participants during the experiment.
5. Cell phones and other electronic devices (**except for calculator functions**) are **not allowed**.
6. Today's session, i.e., Stage 1, will last about **two hours**.

This concludes the general instructions. Please go through the subsequent instructions by yourself and make your decisions carefully. Please raise your hand if you have questions.

IMPORTANT: *To participate in this study, you should have received a **confirmation message** from B2ESS Admin. Should this not be the case for you, please raise your hand.*

¹ Our experiment is a comprehensive study of decision making, comprising of 30 incentivized tasks together with cognitive ability tests. In this Appendix, we show only those tasks relevant to this current paper.

SET A – Individual Decision Making

GENERAL INSTRUCTIONS

This set comprises **19 decision sheets**. The **first 16** sheets are of the form illustrated in the table below.

	Option A	Option B	Decision
1	A	B1	A <input type="checkbox"/> B <input type="checkbox"/>
2	A	B2	A <input type="checkbox"/> B <input type="checkbox"/>
3	A	B3	A <input type="checkbox"/> B <input type="checkbox"/>
4	A	B4	A <input type="checkbox"/> B <input type="checkbox"/>
5	A	B5	A <input type="checkbox"/> B <input type="checkbox"/>
6	A	B6	A <input type="checkbox"/> B <input type="checkbox"/>
7	A	B7	A <input type="checkbox"/> B <input type="checkbox"/>
8	A	B8	A <input type="checkbox"/> B <input type="checkbox"/>
9	A	B9	A <input type="checkbox"/> B <input type="checkbox"/>
10	A	B10	A <input type="checkbox"/> B <input type="checkbox"/>

Each such table lists **10 choices** to be made between a fixed **Option A** and 10 different **Option B's** arranged in an **ascending manner in terms of value** either in the **amount of money** (Decision Sheets A1 – A13) or in the **probability of receiving a higher money outcome** (Decision Sheets A14 – A16). For each row, you are asked to **indicate your choice** in the final “Decision” column – A or B – with a tick (✓).

Decision Sheets **A17 and A18** each involves **one choice**. The last Decision Sheet (**A19**) involves **20 choices**.

Selection of decision sheet to be implemented: One out of the first 18 Decision Sheets (*selected randomly by you*) will be implemented. Should the chosen sheet be from the first 16 decision sheets, **one of your 10 choices** will be further selected randomly and implemented. For **Decision Sheet A19**, one participant in the room will be randomly selected at the end of today's study and **one of his/her 20 choices** will be randomly selected and implemented.

You may now begin.

At any time during the study, should you have questions, please raise your hand. An experimenter will come to you and answer your questions individually.

DECISION SHEET A1

This situation involves your guessing the color – red or black – of a card drawn randomly from a deck of 20 cards, comprising **10 black** cards and **10 red** cards.

Option A: You guess the color – black or red – and then draw a card from the deck of 20 cards. If you make a correct guess, you receive \$60; otherwise, you receive nothing. That is: **50% chance of receiving \$60 and 50% chance of receiving \$0.**

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	50% of receiving \$60, 50% of receiving \$0	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	50% of receiving \$60, 50% of receiving \$0	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	50% of receiving \$60, 50% of receiving \$0	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	50% of receiving \$60, 50% of receiving \$0	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	50% of receiving \$60, 50% of receiving \$0	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	50% of receiving \$60, 50% of receiving \$0	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	50% of receiving \$60, 50% of receiving \$0	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	50% of receiving \$60, 50% of receiving \$0	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	50% of receiving \$60, 50% of receiving \$0	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	50% of receiving \$60, 50% of receiving \$0	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>

DECISION SHEET A6

This situation involves your drawing randomly one card from a **deck of 20 cards with unknown proportions of red and black cards**.

Option A: Guess the color of a card to be drawn randomly by you from a deck of 20 cards with unknown proportions of red and black cards. You will **receive \$60 if your guess is correct**; and **receive \$0 otherwise**.

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	Betting on the color of a card drawn	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	Betting on the color of a card drawn	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	Betting on the color of a card drawn	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	Betting on the color of a card drawn	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	Betting on the color of a card drawn	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	Betting on the color of a card drawn	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	Betting on the color of a card drawn	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	Betting on the color of a card drawn	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	Betting on the color of a card drawn	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	Betting on the color of a card drawn	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>

DECISION SHEET A7

This situation involves your drawing randomly one card from a deck of 20 cards with unknown proportions of red and black cards.

Option A: Guess the color of a card drawn randomly from a deck of 20 cards with unknown proportions of red and black cards. **The deck has at least 5 red cards and at least 5 black cards.** You will **receive \$60 if your guess is correct; and receive nothing otherwise.**

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	Betting on the color of a card drawn	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	Betting on the color of a card drawn	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	Betting on the color of a card drawn	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	Betting on the color of a card drawn	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	Betting on the color of a card drawn	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	Betting on the color of a card drawn	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	Betting on the color of a card drawn	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	Betting on the color of a card drawn	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	Betting on the color of a card drawn	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	Betting on the color of a card drawn	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>

DECISION SHEET A8

This situation involves your drawing randomly one card from a deck of 20 cards **with unknown proportions of red and black cards**.

Option A: Guess the color of a card drawn randomly from a deck of 20 cards with unknown proportions of red and black cards. The deck has **either at least 15 black cards** (not more than 5 red cards) **or at least 15 red cards** (not more than 5 black cards). You will **receive \$60 if your guess is correct; and \$0 otherwise**.

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows in the table below, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	Betting on the color of a card drawn	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	Betting on the color of a card drawn	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	Betting on the color of a card drawn	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	Betting on the color of a card drawn	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	Betting on the color of a card drawn	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	Betting on the color of a card drawn	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	Betting on the color of a card drawn	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	Betting on the color of a card drawn	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	Betting on the color of a card drawn	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	Betting on the color of a card drawn	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>

DECISION SHEET A9

This situation involves drawing a card from a **deck of 20 cards containing black and red cards**. The number of red cards is to be determined by your drawing randomly a ticket from an envelope containing 21 tickets numbered 0 to 20. The number drawn will determine the number of red cards in the deck. **Before** drawing the numbered ticket, you have the following options.

Option A: Guess the color of a card to be drawn randomly by you from **the deck of 20 cards to be constructed by you as described above**. You **will receive \$60 if your guess is correct**; and **receive \$0 otherwise**.

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	Betting on the color of a card drawn	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	Betting on the color of a card drawn	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	Betting on the color of a card drawn	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	Betting on the color of a card drawn	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	Betting on the color of a card drawn	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	Betting on the color of a card drawn	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	Betting on the color of a card drawn	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	Betting on the color of a card drawn	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	Betting on the color of a card drawn	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	Betting on the color of a card drawn	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>

DECISION SHEET A10

This situation involves your drawing a card randomly from a **deck of 20 cards** containing red and black cards. The composition of this card deck is determined as follows. You draw one ticket from an envelope containing 8 tickets numbered 1 to 8. **If the ticket drawn is 1 to 5, then the deck will have 16 red cards and 4 black cards.** Otherwise, **all 20 cards in the deck will have be black cards.**

Option A: Draw a card from the deck of cards constructed in the above described manner. If you **draw a red card, you receive \$60. Otherwise, you receive \$0.**

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	Betting on the cards	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	Betting on the cards	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	Betting on the cards	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	Betting on the cards	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	Betting on the cards	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	Betting on the cards	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	Betting on the cards	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	Betting on the cards	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	Betting on the cards	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	Betting on the cards	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>

DECISION SHEET A12

This situation involves guessing whether the trailing digit (*to the first decimal place*) of the highest temperature (°C) on a historical date in **Singapore** was **odd or even**.

The **historical date** is:

The temperature in Singapore for this historical date is based on the temperatures reported in <http://www.tutiempo.net/en/Climate/> to the first decimal place.

Option A: Guess whether the trailing digit (*to the first decimal place*) of the highest temperature (°C) on the above **historical date** in **Singapore** was odd or even. You **receive \$60 if your guess is correct**; and nothing otherwise.

Please write down **your guess**: odd even .

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	Betting on Singapore Temperature	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	Betting on Singapore Temperature	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	Betting on Singapore Temperature	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	Betting on Singapore Temperature	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	Betting on Singapore Temperature	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	Betting on Singapore Temperature	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	Betting on Singapore Temperature	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	Betting on Singapore Temperature	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	Betting on Singapore Temperature	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	Betting on Singapore Temperature	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>

DECISION SHEET A13

This situation involves guessing whether the trailing digit (*to the first decimal place*) of the highest temperature (°C) on a historical date in Istanbul was **odd or even**.

The historical date is:

The temperature in Istanbul for this historical date is based on the temperatures reported in <http://www.tutiempo.net/en/Climate/> to the first decimal place.

Option A: Guess whether the trailing digit (*to the first decimal place*) of the highest temperature (°C) on the above **historical date** in **Istanbul** was odd or even. You **receive \$60 if your guess is correct**; and nothing otherwise.

Please write down **your guess**: odd even .

The **Option B** column lists **10 amounts** (*displayed in an ascending manner*) each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (✓).

	Option A	Option B	Decision
1	Betting on Istanbul Temperature	Receiving \$15 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
2	Betting on Istanbul Temperature	Receiving \$19 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
3	Betting on Istanbul Temperature	Receiving \$23 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
4	Betting on Istanbul Temperature	Receiving \$25 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
5	Betting on Istanbul Temperature	Receiving \$27 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
6	Betting on Istanbul Temperature	Receiving \$29 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
7	Betting on Istanbul Temperature	Receiving \$30 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
8	Betting on Istanbul Temperature	Receiving \$31 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
9	Betting on Istanbul Temperature	Receiving \$33 for sure	A <input type="checkbox"/> B <input type="checkbox"/>
10	Betting on Istanbul Temperature	Receiving \$35 for sure	A <input type="checkbox"/> B <input type="checkbox"/>